Acta Mechanica Sinica » 2011, Vol. 27 » Issue (6) :866-876   DOI: 10.1007/s10409-011-0526-9

A note on the definition of fractional derivatives applied in rheology

Fan Yang, Ke-Qin Zhu

Department of Engineering Mechanics, Tsinghua University, 100084 Beijing, China

Abstract It is known that there exist obivious differences between the two most commonly used definitions of fractional derivatives—Riemann – Liouville (R – L) definition and Caputo definition. The multiple definitions of fractional derivatives in fractional calculus have hindered the application of fractional calculus in rheology. In this paper, we clarify that the R – L definition and Caputo definition are both rheologically imperfect with the help of mechanical analogues of the fractional element model (Scott – Blair model). We also clarify that to make them perfect rheologically, the lower terminals of both definitions should be put to $-\infty$. We further prove that the R – L definition with lower terminal $a \to -\infty$ and the Caputo definition with lower terminal $a \to -\infty$ are equivalent in the differentiation of functions that are smooth enough and functions that have finite number of singular points. Thus we can define the fractional derivatives in rheology as the R – L derivatives with lower terminal $a \to -\infty$ (or, equivalently, the Caputo derivatives with lower terminal $a \to -\infty$) not only for steady-state processes, but also for transient processes. Based on the above definition, the problems of composition rules of fractional operators and the initial conditions for fractional differential equations are discussed, respectively. As an example we study a fractional oscillator with Scott – Blair model and give an exact solution of this equation under given initial conditions.

Keywords: Fractional derivatives   Rheology   Riemann – Liouville definition   Caputo definition

Received 2010-07-20; published 2011-09-13

Fund:

The project was supported by the National Natural Science Foundation of China (10972117).

Cite this article:

Fan Yang, Ke-Qin Zhu.A note on the definition of fractional derivatives applied in rheology[J] Acta Mechanica Sinica, 2011,V27(6): 866-876