AN INTEGRATED MODEL FOR FUZZY TOPOLOGY AND QUALITATIVE DISTANCE FOR DISCRETE SPACE

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ABSTRACT:

The topological relationship of disjoint is ubiquitous in physical world, while the traditional topological relationship models do not take more considerations about it. Point set topology is exhaustive for topological relation set and rational for topological computation; nevertheless, it is not good at spatial reasoning. Although RCC5 or RCC8 is competent for spatial reasoning, it is not fit for discrete space. So the ultimate aim of this paper is to work out a topological model meeting for representing and reasoning spatial entities in discrete space, which extended the RCC5 model with the relationship of parthood and integrated the qualitative distance into topological relation representation. For this, we fuzzed the discrete crisp region with k-neighborhood constructed by Voronoi diagram, so that we can use the tri-tuple {P(X,Y)，P(X, $\neg Y$)，P(Y,X)} to represent the topological relations, and k-neighborhood can imply the qualitative distance. So we gained a plentiful semantic model for spatial topological relation and qualitative distance relation representation.

1. INTRODUCTION

Qualitative spatial reasoning has been paid more and more attentions. Nowadays, its applications have become the focus of spatial reasoning in GIS, robot navigation, computer vision and the interpretation of natural language (Laguna 1992, Frank 1996). The qualitative presentation and reasoning of spatial relationships is vital for the field of qualitative spatial reasoning in GIS, which includes spatial topological relationships and metric relationships. In this paper we center upon the research of topological and distance relationships. Both relationships imply a kind of spatial constraint, which determined by the objects' geometric characters. Topological relationship is a relation that is invariant under homeomorphisms, which is significance for GIS modeling, analyzing and reasoning. How to identify the topological relationships between spatial objects is a critical point in GIS. During recent years, topological relationships have been investigated extensively and gained many achievements. There are three clues for topological relationships: Allen et al (1983) identified thirteen topological relationships between two temporal intervals; Egenhofer et al (1994) proposed the 4-intersection and 9-intersection model based on point set; eight topological relationships were derived from RCC theory based on classic logic (Randell et al 1992). The 9-intersection approach decomposed the object into the interior A^o, boundary ∂A and exterior A^-, and the topological relationships can be concluded from the 3*3 intersection matrix making up of the three part of both objects. The eight topological relationships between two simple regions (no hole and connected), thirty-three line-line topological relationships and nineteen point-line topological relationships have been verified. The exhaustive and rationality of the model have been consensus (Cobb 1995). But for region-region topological relationships the definition is loosely, as a consequence many relations defined by the model are rare even unviable for naive geographical world. On the contrary, the disjoint relations, which are plenty of in physical world, are unable to discriminate well (Cao 2001), so the 9-intersection model need other approach to remedy for representation disjoint relations. Since the model is based on the point set theory which including vast complicated computations on set, the abilities of spatial reasoning are limited. This situation has been mended partly for the composition table of topological relation developed by Egenhofer et al, whereas it is still inaptitude for the qualitative reasoning with incomplete information. The thirteen temporal relationships can be extended to distinguish 13*13 topological relationships between two objects, which have the merit of simplify reasoning computation and convenience. However, the directly dimensional extension brings out the loss of some relationships (Cao 2001). So the model is not a formal model. Randell et al claim that the topological relationship can been deduced based on logic. Eight topological relationships were identified based on their RCC theory. Subsequently, Cohn et al (1997) adopted the theory for spatial reasoning and improved its JEPD (Joint Exhaustive Pair Disjoint) that is necessary for spatial reasoning model. Unfortunately, the RCC8 or RCC5 is competent for continuous space but not suitable for discrete phenomena. For metric relation, the research is focus on qualitative distance relations by now. Reasoning of distance relation is based on the spatial cognition, and the approach of reasoning need for avoiding complex computation. The goal is to furnish people with query and reasoning engine among disperse objects, which is conform with human cognition in naïve geographic space. So, it is urgent for a represent and reasoning model in discrete space, which appease for spatial reasoning and cognition.

In this paper, we design a topological relation model based on deeply analysing the mechanisms of the 9-intersection approach and RCC5 theory. The tri-tuple（P(A,B),P(A, ┐B),P(B,A)）is

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developed with the primitive operator of parthood (P) and the complement (┐) of regions in the model. Beside for borrowing of the logic rationality of the RCC theory, the qualitative distance relation is introduced into the topological model to remedy for the limitation of 9-intersection approach for disjoint relationships by conversation the disperse geographic phenomena into continuous space with the region's Voronoi diagram. For the remainder of this paper, Section 2 introduces the RCC theory and extends the RCC5 model based on the parthood and the complement of the region. Section 3 proposes the topological relation model in discrete space based on fuzzy region. Section 4 discusses the reasoning of topological relation in discrete space. In section 5 we conclude the paper and point out the directions for future research.

2. EXTENSION OF RCC5

Region is considered as the spatial primitive in RCC5, which replace the point set with the set of regions. There are some debates about whether the region or the point should be recognized as the primitive. In the domain of GIS, the conventional GIS is based on the crisp set theory and the basic spatial operations ground on the computation geometry and classical algebra, so it is reasonable for selection point with precise position as the spatial primitive. On the other hand, the qualitative spatial reasoning emphasizes on the cognitive rationality of spatial representation, with that point of view, the regional phenomena are all over the world such as lakes, cordillera even rivers and roads, so they claim it is certainly reasonable. We think that regions are the components of physical world, while points are the results of human abstracting the world, which is complicated. In addition, the field of qualitative spatial reasoning is the geographic space that accords with human's commonsense, which is intuitional and approximately, so the region is adopted as the spatial primitive in this paper.

2.1 Topological Relationships Representation with RCC5

RCC theory is closely based on the Clarke's system, so it is necessary to briefly introduce some fundamental notations. Clarke presented an extended account of logic axiomatization for a region-based spatial calculus. He defined the dyadic relation of C(X,Y) (means X connect with Y) as the basic operator of the system, where $C(X,Y) \equiv def \exists x \in X, \exists y \in Y \|x - y\| = 0$, following that the five topological relation predicates of DC、PO，PP，PPi and '=' are identified.

The RCC theory predefined that any region has an untangential interior, which guarantees the space to be dividable infinitely, what is to say that the region is non empty and continuous. In order to represent the points, lines and polygons used in conventional GIS, We must extend the RCC system.

2.2 The Extended RCC Space

The classical GIS abstractly depicts the geographic world with point, line and polygon, meanwhile the RCC system requests all the objects to be regional like. In order to using RCC to represent those entities, we must revise the notation of region firstly. Different from traditional region, three discriminated notations of region called o-dimension region, 1-dimension region and 2-dimension region (denoted as $\dot{R}$, $\widetilde{R}$ and R separately) are defined as the following:

Definition 1: supposing U is a geographic space, for any geographic object x, x∈U, if A(x)=0 ∧Perim(x)=0 then x∈ $\dot{R}$ Here A(x) and Perim(x) are the object's area and perimeter. Obviously both the area and perimeter of an object are zero, which means that the object is a traditional point. The object with 0-area and 0-perimeter may not be existed in the physical world, but it is important for the conceptual world, just as the number 0 is vital for mathematics development.

Def 2: object x∈U, if A(x)=0 ∧ Perim(x)≠0 then x∈ $\widetilde{R}$

From the definition, we can found the $\widetilde{R}$ is just like the traditional line object.

Def 3: object x∈U, if A(x) ≠0 ∧ Perim(x)≠0 then x∈R This R is just like the traditional region.

Def 4: the geographic space which composed of the $\dot{R}$, $\widetilde{R}$ and R is called the extended RCC space denoted by U^r. that is: $U^r = \vee_i \dot{R} \cup \vee_i \widetilde{R} \cup \vee_i R$

With these definitions, the following proposition exist for U^r:

Proposition1: geographic space

$$U \subset U^r \to \phi, X \in U^r, \forall A_i \in U^r \to \vee_i A_i \in U^r, \forall U, V \in U^r \to U \wedge V \in U^r$$

That is to say, U^r is a topological space. On the basis of U^r, we can represent the whole geographic space with RCC model.

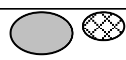
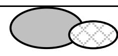
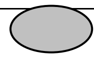
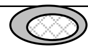
2.3 Topological Representation Based on P*(X, Y)*

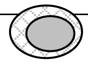
Based on the spatial primitive of region, this paper revises the basic operator of *C(X, Y)*. In order to use the existed function of conventional GIS to represent and reasoning spatial objects, the operator of *C(X, Y)* is replaced with the relation of *P(X, Y)* (called as X is part of Y). *P(X, Y)* can be defined as following:

Definition 5: $P(X,Y) \equiv def[\ \forall x \in X \to x \in Y\ ]$

From the definition we can found that the *P(X, Y)* just like the operator of Contain*(Y, X)*, which is available in the conventional GIS, so we can implement the spatial representation and reasoning in the existed GIS tools.

On the base of the operator *P(X, Y)*, we are able to depict the five relation of RCC5(DC, PO, =, PP, PPi) with the tri-tuple (*P(X, Y)*，*P(X, ¬Y)*，*P(Y, X)*), where $\neg Y$ is the complement of region Y. The result lists in the table1.

Tab.1 the model of RCC5 based on P(X, Y)RCC5		Tri-tuple of P			Figure
Predicate	Semantics	P(A,B)	P(A, $\neg B$)	P(B,A)	
DC	Disjoint	F	T	F	
PO	Intersection	F	F	F	
=	Equal	T	F	T	
PP	Contain	T	F	F	

PPi	ContainedBy	F	F	T	

Using the dyadic logic to express the result of the *P(A, B)* and *P(A, ¬B)*, there are 2^3=8 topological relationships identified by them, however, some situations depicted by these tuples (T, T, T), (T, T, F) and (F, T, T) are impossible for the real world.

For the extended RCC space, we can conclude the possible topological relationships between two regions considering the characters of 0-dimension, 1-dimension and 2-dimension region. The results list in table2.

Tab.2 the topological relations between the regions of RCC

	0-dimension region $\dot{R}$	1-dimension region $\tilde{R}$	2-dimension region R
0-dimension region $\dot{R}$	DC、=	DC、PP	DC、PP
1-dimension region $\tilde{R}$	DC、PPi	DC、PO、=、PP、PPi	DC、PO、PP
2-dimension region R	DC、PPi	DC、PO、PPi	DC、PO、=、PP、PPi

3. TOPOLOGICAL RELATION MODEL FOR DISCRETE SPACE

From the analysis in section 2.3, we can see that the tri-tuple is competent for representation of the topological relationships in RCC space. Following the approach of the extension from RCC5 to RCC8, we can extend the dyadic logic (TRUE and FALSE) to ternary logic (TRUE, FALSE and MAYBE), so that the tri-tuple can discriminate 3^3=27 topological relationships. Certainly, the different of the definition of TRUE, FALSE and MAYBE will result in variant set of topological relationships; we make use of the following definitions in this paper.

$$P'(X,Y)=\begin{cases} T & P(X,Y')=T \\ M & P(X,Y)=T \wedge P(X,Y')F \\ F & P(X,Y')=F \end{cases}$$

$$P'(X,\neg Y)=\begin{cases} T & P(X,\neg Y)=T \\ M & P(X,\neg Y)=F \wedge P(X,\neg Y')=T \\ F & P(X,\neg Y)=F \end{cases}$$

Where *P'(X, Y)* is the operator of parthood by using ternary logic; Y° is the interior of the region Y, and ¬ Y° is the complement of the interior of region Y.

By comparing every situations identified by the model, we can found that there only eight situations are possible in physical world, which are just as the RCC8; illustrated as the figure 1.

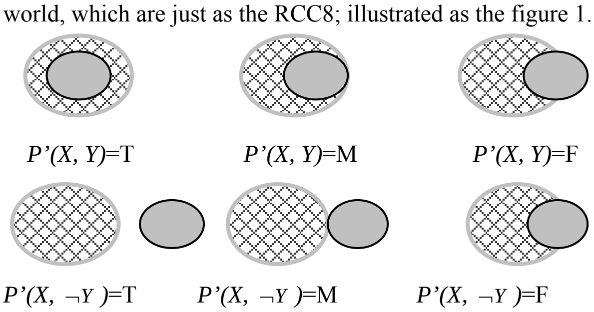


Fig.1 topological relations based on ternary logic of *P(X, Y)*

3.1 Fuzzy Region

We can use tri-tuple to represent the topological relationships in RCC8 by means of extension dyadic logic to ternary logic. Meanwhile, we can extend the region from crisp to fuzzy, so that a continuous space can be formed with distributed geographic phenomena.

According to the Decompose Theorem of fuzzy set, supposing $\underset{\sim}{A} \subset \tilde{f}(x)$, $\tilde{f}(x)$ is a fuzzy set, A_λ is the λ cut set of $\underset{\sim}{A}$, so, we can get that $\underset{\sim}{A} = \bigcup_{\lambda\in[0,1]} \lambda A_\lambda$, and the membership function of λA_λ is:

$$\mu_{\lambda A_\lambda}(x)=\begin{cases} \lambda & x \in A_\lambda \\ 0 & x \notin A_\lambda \end{cases}$$

Here A_λ is a crisp set, and A_1 is the core of the fuzzy set $\underset{\sim}{A}$.

When discretized the real number of λ with integer, we can get the notation of fuzzy region as following:

Definition 6: the fuzzy region $\underset{\sim}{A}'$ of crisp region A is composed of the core and the definite crisp regions (λ cut regions), $\underset{\sim}{A}' = \bigcup_{\lambda_i \in \lfloor 0,1 \rfloor, i\in N} \lambda_i A_{\lambda_i}$,which is to say that fuzzy region is a set of encircled crisp region. It can be illustrated as the figure 2.

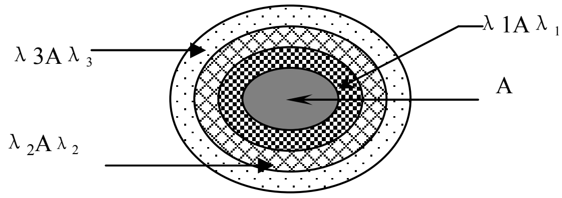


Fig.2 fuzzy region

Here λ_i is assigned with {1, 2, 3} and $\lambda_1 > \lambda_2 > \lambda_3$. According to real situations, i can be assigned much bigger. Based on the discussion, the following propositions can be deduced:

Proposition 2： $\forall i, j \in N \;\; i < j \rightarrow P(\lambda_i A_{\lambda_i}, \lambda_j A_{\lambda_j}) = T$

Proposition 3 ： $a \in R$, $\forall i, j \in N$, i<j, $P(a, \lambda_i A_{\lambda_i}) = T \rightarrow P(a, \lambda_j A_{\lambda_j}) = T$

3.2 Fuzzy Region Construction Based on k-neighborhood

The topological relationship of disjoint is ubiquitous in physical world; meanwhile, the traditional topological relationship model does not take more considerations about it, which is a severe vice for commonsense reasoning. This paper integrates the qualitative distance into topological relation representation to remedy for the flaws of traditional topological model. Many experts have paid attentions to qualitative distance relation (Frank 1992, Renz 2002), but the focus is how to transform quantitative distance into qualitative distance relations. The semantics of qualitative distance relations are complex, which are related with the objects' size, figure, correlative position with associated objects and the reference frame (Frank 1992). Voronoi diagram identifies the qualitative distance relation with no misinterpretations, which can consider the objects' size, position and the other related surroundings. So the k-neighborhood is available for construction the topological model integrated with distance.

3.3 Topological Relation Model Based on Fuzzy Region

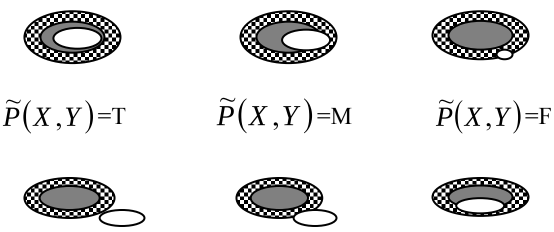
According to the classification of qualitative distance, we can correspondingly construct the k-neighborhood, for example the qualitative distance is graded into far, quite-far and close, then we can construct 3-neighborhood. For simplified the description, we use the 1-neighborhood to construct fuzzy region $\underset{\sim}{A}'$. Then the TRUE, MAYBE and FALSE can be redefined as following:

$$\widetilde{P}(X,Y)=\begin{cases} T & P(X,Y_{\perp})=T \\ M & P(X,Y_{T})=T \wedge P(X,Y_{\perp})=F \\ F & P(X,Y_{T})=F \end{cases}$$

$$\widetilde{P}(X,\neg Y)=\begin{cases} T & P(X,\neg Y_{T})=T \\ M & P(X,\neg Y_{T})=T \wedge P(X,\vee Y_{\perp})=F \\ F & P(X,\neg Y\perp)=F \end{cases}$$

$$\widetilde{P}(Y,X)=\begin{cases} T & P(Y_{T},X)=T \\ M & P(Y_{\perp},X)=T \wedge P(Y_{T},X)=F \\ F & P(Y_{\perp},X)=F \end{cases}$$

Here X is crisp region; Y is fuzzy region formed with 1-neighborhood; $Y_{\perp}$ is the upermium of fuzzy region $\underset{\sim}{A}'$, which just is the core of $\underset{\sim}{A}'$. Y_{T} is infimum, so the parthood can be illustrated as figure 3:



$\widetilde{P}(X,Y)$=T $\widetilde{P}(X,Y)$=M $\widetilde{P}(X,Y)$=F

$\widetilde{P}(X,\neg Y)$=T $\widetilde{P}(X,\neg Y)$=M $\widetilde{P}(X,\neg Y)$=F

Fig.3. the part relation of tri-tuple based on fuzzy region

Based on the operator $\widetilde{P}(X,Y)$, we can use the tri-tuple { $\widetilde{P}(X,Y)$, $\widetilde{P}(X,\neg Y)$, $\widetilde{P}(Y,X)$)} to identify the topological relationship between crisp region and fuzzy region formed with 1-neighborhood. The results list in table 3.

By analysing the tri-tuple, we can see that 3^3=27 topological relationships can be discriminated theoretically. Considering the definition 6, proposition 2 and proposition3, we can deduce that some situations are impossible such as （T，T，T），（T，M，T）, and only ten cases listed in table 3 are valid.

Tab.3 topological relation between crisp and fuzzy regions

Predicate	Semantics	$\widetilde{P}(X,Y)$	$\widetilde{P}(X,\neg Y)$	$\widetilde{P}(Y,X)$	Figure
DD	Disjoint	F	T	F	
DO	k-1Disjoint& k Intersection	F	M	F	
DP	k-1Disjoint & k Contain	M	M	F	
OP	k-1Intersect & k Contain	M	F	F	
OO	Disjoint	F	F	F	
EP	k-1Equal & k Contain	T	F	M	
PP	Contain	T	F	F	
IP	k-1ContainBy k Contain	M	F	M	
IE	k-1Containby & k Equal	M	F	T	
II	ContainBy	F	F	T	

On the above, we only interpret the representation of topological relationships between fuzzy region formed with 1-neighborhood, in the same way, we can substitute ternary logic for many-value logic, and the 1-neighborhood replaced with n-neighborhood accordingly, so that the qualitative distance can distinguish into more grades.

4. 4 REASONING OF TOPOLOGICAL RELATION IN DISCRETE SPACE

The basic approach of spatial reasoning is based on constraint (Frank 1996, Cohn 1995,2001), which need define the set of fundamental spatial topological relationships firstly, and then deduce the composition table of topological relationships. This reasoning approach presumes that any topological relationship is composed of the basic relationships, so that the set of basic relationships must be Joint Exhaustive and Pair Disjoint (JEPD).

4.1 Explain of the Model's JEPD

There are ten basic topological relationships between the crisp region and fuzzy region constructed with 1-neighborhood (as listed in tab3). From representation procedure we can see the Pair Disjoint is obvious (any pair of relationships' tri-tuple is dissimilar).

For the property of Joint Exhaustive, we can get interpretations from the RCC5 model. From definition 6 we can conclude that the fuzzy region $\underset{\sim}{A}'$ is the family of crisp region $\lambda_i A_{\lambda i}$. In the light of the basic principle of Mereology, the fuzzy region $\underset{\sim}{A}'$ can be decomposed into n components, correspondingly, the computation of topological relationship can be transformed into inferring the topological relationship between crisp region R and $\lambda_i A_{\lambda i}$. For the topological relationships of the crisp region and the fuzzy region constructed with 1-neighborhood, the result can be gotten by the intersection of crisp region R with both the core A of $\underset{\sim}{A}'$ and the 1-neighborhood $\lambda_1 A_{\lambda 1}$ of A. By using of RCC5, there are five basic relations for each element of the intersection. Additionally, proposition 1 shows that A is part of $\lambda_1 A_{\lambda 1}$, so the topological relationship between R and $\underset{\sim}{A}'$ can be expressed as following:

$$a\in R, b\in \underset{\sim}{A}', b=b_0 \underset{\sim}{\cup} b_1, P(b_0,b_1)=T \wedge P\left(\neg b_1,\neg b_0\right)=T$$

$$Topo(a,b)=Topo(a,b_0)\circ Topo(a,b_1)$$

Here b_0 and b_1 are the core and the 1-neighborhood of the region R separately; Topo(a, b) is the topological relationship between region a and region b, and "∘" is the composite operator of topological relationships. Based on the composite table of RCC5 combined with proposition 1 and 2, we can conclude the ten basic relationships between crisp region and fuzzy region listed in table 3.

4.2 Predicates of Topological Relationships and Its Reasoning

The topological model proposed in this paper can be regard as the extended form of RCC5 in discrete space, which substitutes the C(X, Y) for P(X, Y). Consequently, we can get the predicates of this topological model by extending the basic predicates of RCC5. Synthesizing with the character of the fuzzy region, we define ten basic topological relationship predicates as listed in table 3. The set of topological relationship predicates is diverse with the different of k values of k-neighborhood. Each topological predicate consists of k+1 basic predicates of RCC, and DC, PO, =, PP and PPi are simplified with D, O, E, P and I in short. From the denotation of the predicate, we can infer two sides information for example DO(A, B) show that the fuzzy region is formed with 1-neighborhood and A is disjoint from B, A intersects with the 1-neighborhood of B.

Corresponding with RCC5's logic reasoning approach by using axiomatic topological properties and the C(X,Y) operator, this model also can adopt the operator of P(X,Y) and the properties of $P(X,A)=T \wedge P(A,Y)=T \rightarrow P(X,Y)=T$ for reasoning and computation the tri-tuple{ $\widetilde{P}(X,Y)$, $\widetilde{P}(X,\neg Y)$, $\widetilde{P}$ (Y, X)}. About the detail process of reasoning with this topological model, we will discuss in another paper for the limitation of this paper's topic. But we design the concept diagram of the set of basic topological relationships grounded on the diagram of RCC5, which easy to reason and predict the topological relationships between moving objects, which illustrated in figure 4.

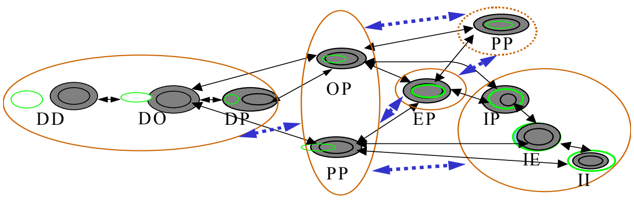


Fig 4. Conceptual neighbour diagram of topologic relations

From all above, we can see that the semantics of this topological model is plentiful, which represent not only the topological relationships but also qualitative distance by using the k-neighborhood replace with the object itself. At the same time, this model makes it easy to encode these topological relationships in application systems so that we can reason and query with spatial topological relationships recurring to computers.

5. DISCUSSION AND CONCLUSION

This paper first reviews the existed topological model and analyzes their merits and shortcomings for representing the topological relationships. Point set topology is exhaustive for topological relation set and rational for topological computation; nevertheless, it is not good at dealing with qualitative spatial reasoning. Although RCC5 or RCC8 has the abilities of spatial reasoning and management of uncertainty, it is not fit for discrete space. In order to represent and reason the topological relationships in discrete space, we propose a topological relationship model for discrete space based on fuzzy region. To illustrate this model, our investigations develop from following aspects:

①Make region for the spatial primitive of spatial reasoning, and construct a topological space based on the definitions of 0-dimension region, 1-dimension region and 2-dimension region, which realize to represent the naive geographic world with regions and integrate with traditional GIS.

②Take *P(X, Y)* as the basic spatial operator, and define the tri-tuple {$P(X,Y)$, $P(X,\neg Y)$, $P(Y,X)$} to identify the topological relationships between spatial objects, which extends the RCC5 to fit for traditional GIS tools.

③Based on the fuzzy region formed with the objects k-neighborhood, we propose a spatial topological relationship model for discrete space and deduce the set of basic topological relationships, which can represent the topological relationships between discrete objects and integrate the qualities distance into topological relation.

However, the representation and reasoning of spatial relation is a complex subject respected with many domains. Although we have adopted the RCC for discrete space, it is only the jumping-off point for spatial reasoning and representation in discrete space. Our future studies will be focused on these subjects:

①Based on the ontology of this paper proposed, study a consistent representation and reasoning model integrated with topological, directional and distant relationships.

②Study the representation and reasoning of spatio-temporal relationships between moving objects.

③Research the applications of this model and modify it according with the domain of applications.

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