ANALYZING ON PIXEL POSITIONING ACCURACY OF SAR IMAGES BASED ON R-D LOCATION MODEL

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ABSTRACT:

The range-Doppler (R-D) model, which requires no reference points and is independent of satellite attitude knowledge and control, is used to determine the location of SAR images. However, its error has not been analyzed sufficiently. This paper investigates the effects of the pixel positioning accuracy of SAR images based on R-D model on uncertain measurement errors such as range measurements, Doppler center frequency measurement, orbit and the earth model. The test results indicate the earth model and zero-Doppler condition have severe effect on the pixel positioning accuracy, but they can be corrected accurately using the known DEM and Doppler parameters. Orbit errors are the critical errors and it must be corrected using the precise orbits to achieve a positioning accuracy of around 25 m. Compared with the errors given above the atmospheric delay error is less, but it may become remarkable in low latitude areas. Moreover, the pixel positioning errors of SAR images are nonlinear, and the errors decrease gradually with decreasing latitude as well as increasing longitude.

1. INTRODUCTION

Effective utilization of synthetic-aperture radar (SAR) image often requires precise location of each image pixel. Historically, the absolute positioning of space-borne SAR image has relied on the use of reference points on the earth which are recognizable in SAR image. However, for many oceanic applications, such as registration of image to monitor ice motion, the image dose not contain any recognizable features, therefore, no reliable reference points are available for image positioning. In this conditions, the range-Doppler model was presented, which has an advantage over previous techniques in that it requires no reference points and is independent of satellite attitude knowledge and control.

The R-D model was originally presented by Borwn (Brown, 1981). It requires only the satellite ephemeris data and the characteristics of the SAR data collection system as input. The accuracy of the result depends primarily on the validity of the assumed earth model and the characterization of the target range, the satellite ephemeris data and the center Doppler information. Many methods were developed to solve R-D equations, however the error of it has not been analyzed sufficiently (Curlander, 1982; Chen, et al, 2000; Zhou, et al, 2001; Zhang, et al, 2005; Zou, 2005; Yang, et al, 2006). In this paper, the three fundamental relationships of R-D model are introduced and analyzed firstly and then its error models are derived based on the law of error propagation by differentiating the equations with respect to the slant range from sensor to target, the satellite positions and velocities, the target positions, the Doppler center frequency and the normal height. At last, three scenes test data of the ERS-2 C-band SAR from Tibet Zhang autonomous region, Nanjing city and Taiwan province are used to investigate the effects of the pixel positioning accuracy of SAR images on uncertain measurement errors such as range measurement, Doppler center frequency measurement, satellite orbit and the earth model.

2. DESCRIPTION OF THE R-D MODEL

The location of an arbitrary pixel in a SAR image is determined by the intersection of the centroid of the radar beam with the planet surface. This intersection is determined by three fundamental relationships: 1) the SAR range equation defining the distance from the sensor to the target; 2) the SAR Doppler equation defining the plane of the centroid; 3) a model describing the earth's shape. Figure 1 illustrates the intersection of three equations.
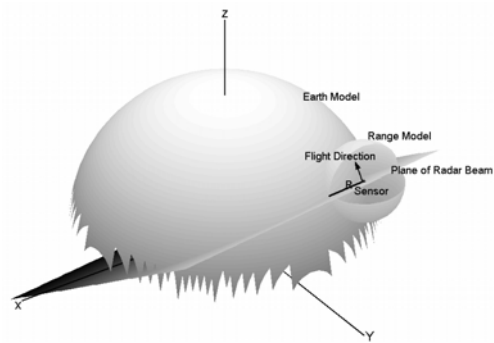

Figure 1. Schematic diagram for pixel positioning using R-D model. R is the slant distance from sensor to target.

2.1 Range equation

The slant range from the sensor to the target on the earth is defined by the equation

$$R = \left[\left(\vec{R_s} - \vec{R_t} \right) \bullet \left(\vec{R_s} - \vec{R_t} \right) \right]^{\frac{1}{2}} \qquad (1)$$

where $\vec{R}_s$ and $\vec{R}_t$ are the satellite and target position vectors, respectively, and " $\bullet$ " is the vector dot product.

2.2 Doppler equation

The return echo data from a point target is shifted in frequency by an amount proportional to the relative velocity between the satellite and target. The equation describing the Doppler shift is given by

$$f_D = \frac{2}{\lambda R}(\vec{V}_s - \vec{V}_t) \bullet (\vec{R}_s - \vec{R}_t) \qquad (2)$$

where f_D is the Doppler frequency associated with the return echo data, λ is the radar wavelength, $\vec{V}_s$ is the satellite velocity vector, $\vec{V}_t$ is the target velocity vector. The WGS84 coordinate system is adopted in this paper, so the $\vec{V}_t$ equals to zero.

The Doppler equation given by Eq. (2) defines the plane of the center of the radar beam at a specific instant in time. The intersection of this plane with the assumed earth model is the locus of potential targets as shown in Fig. 1.

2.3 Earth model equation

The equation describing the approximate shape of earth is given by

$$\frac{x_t^2}{(a+h)^2} + \frac{y_t^2}{(a+h)^2} + \frac{z_t^2}{b^2} = 1 \qquad (3)$$

$$a = 6378.137 km$$
$$b = 6356.752 km$$

where x_t, y_t, z_t are the *3-d* coordinate components of the target position vector $\vec{R}_t$ in the *x, y* and *z* direction, respectively, *a* is the semi-major axis, *b* is the semi-minor axis, and *h* is the mean normal height.

3. ERROR ANALYSIS

3.1 Error models

Base on the error propagation theory, the error models can be derived by differentiating the equations (1)~(3) with respect to

the slant range R, the satellite position $\vec{R}_s$, the target position $\vec{R}_t$, the Doppler frequency f_D, the satellite velocity $\vec{V}_s$ and the mean normal height *h*, yields

$$RdR = \Delta x_{st}\Delta dx_{st} + \Delta y_{st}\Delta dy_{st} + \Delta z_{st}\Delta dz_{st} \qquad (4)$$

$$\frac{\lambda R}{2} df_D = \Delta x_{st}dv_x + \Delta y_{st}dv_y + \Delta z_{st}dv_z$$
$$+ v_x\Delta dx_{st} + v_y\Delta dy_{st} + v_z\Delta dz_{st} \qquad (5)$$

$$(a+h)b^2 dh = x_t b^2 dx_t + y_t b^2 dy_t$$
$$+ z_t(a+h)^2 dz_t \qquad (6)$$

$$\Delta x_{st} = x_s - x_t$$
$$\Delta dx_{st} = dx_s - dx_t$$

where x_s, y_s, z_s and v_x, v_y, v_z are the components of the satellite position $\vec{R}_s$ and velocity $\vec{V}_s$ in the *x, y* and *z* direction, respectively. Δy_{st} and Δz_{st} are the same as Δx_{st}, but for *y* and *z* direction. dx_t, dy_t and dz_t are pixel location uncertainties in *x, y* and *z* direction, respectively. $dx_s, dy_s, dz_s, dv_x, dv_y, dv_z, dR, dh$ and df_d are the uncertainties of the satellite orbit measurements, range measurements caused by atmospheric delay, the normal height measure and the Doppler information, respectively. Δdy_{st} and Δdz_{st} are the same as Δdx_{st}, but for *y* and *z* direction. Using equations (4)~(6), the dx_t, dy_t, dz_t can be expressed as the linear functions of $dx_s, dy_s, dz_s, dv_x, dv_y, dv_z, dR, dh$ and df_d whose coefficients, symboled as $a, b, \cdots, h, i$, can be computed when the initial values, such as *h*, R, $\vec{V}_s$, $\vec{R}_s$ and $\vec{R}_t$, are known.

3.2 Calculation and analysis

Table 1 lists three scenes test data of the ERS-2 C-band SAR from Tibet Zhang autonomous region, Nanjing city and Taiwan province. Based on the three test data, three groups of error coefficients are computed, respectively. Only the first group for Tibet Zhang are given in table 2 due to space constraint in this paper.

Site	R (m)	h (m)	$\vec{R}_s$ ($m \times 10^6$)			$\vec{V}_s$ ($m \times 10^3$/s)			$\vec{R}_t$ ($m \times 10^6$)		
TZ	84927	4827	-0.441	5.808	4.162	1.459	4.398	-5.963	-0.093	5.146	3.760
NJ	84928	18	-3.293	5.152	3.725	-0.594	4.168	-6.271	-2.664	4.707	3.370
TW	84718	759	-3.616	5.491	2.838	-0.201	3.373	-6.758	-2.958	5.021	2.584

Table 1. The parameters of SAR images in three sites

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
x	1.0319	-1.7718	-1.3051	-9.1167	17.3118	10.5423	2.4225	2.1977	-0.6297
y	0.0083	0.5383	-0.3401	-26.9256	51.1289	31.1360	-0.2547	0.5727	-1.8598
z	0.0139	-0.7741	0.4298	36.3311	-68.9891	-42.0122	0.4050	0.9602	2.5094

Table 2. Error propagation coefficients for Tibet Zhang area

From table 1 and 2, we can see that in Tibet Zhang area the mean normal height value is up to 4827m, in where around 10608m, 2764m and 4634m positioning error would be caused in *x, y* and *z* direction respectively if the normal height were not adopted. So the earth model must be corrected using the known DEM. Fortunately, the NASA Shuttle Radar Topographic Mission (SRTM) has provided DEM for over 80% of the globe and the vertical error of the DEM is reported to be less than 16m. When the SRTM DEM in Tibet Zhang area is used, the error caused by elevation can reduce to 35m, 9m and 15m. The Doppler center frequency error is the second error resource to be considered, which will be produced when the Doppler center frequency true value is not equal to zero that is assumed in SAR imaging. Taking the test data in Tibet Zhang area as example, its Doppler center frequency true value is -113.55Hz. If the true value were not adopted, around 71.5m, 211.1m and 284.9m positioning error would be caused in *x, y* and *z* direction, respectively. Fortunately, the true Doppler center frequency can be estimated by the Doppler frequency center and Doppler frequency variation rate parameters which can be obtained from the SAR auxiliary data. When the true Doppler center frequency is adopted the Doppler center frequency nonzero error can be ignored.

Once the earth model error and the Doppler center frequency nonzero error have been corrected, the satellite orbit parameter (positions and velocities in *x, y* and *z* direction) errors become the most dominant. Table 3 shows the typical orbit and atmospheric delay errors in pixel location with the ERS-2 C-band SAR. Since the orbit is very nearly a straight line during one second, the high precision *XYZ* velocity (XYZ_{vol}) can be approximated as follows

$$XYZ_{vol}(t) = XYZ_{pos}(t + 0.5\sec) - XYZ_{pos}(t - 0.5\sec) \quad (7)$$

So the uncertainty of the *XYZ* velocity can be given by $\sqrt{2}$ times the uncertainty of the *XYZ* position.

The calculations in table 3 are based on the nominal system parameters of the ERS-2 and assume the error of the mean normal high is zero and the true value of Doppler center frequency is used.

	Typical error	TZ SAR image (36.299°, 91.105°)			NJ SAR image (32.097°, 119.511°)			TW SAR image (24.031°, 120.509°)		
		dx_t	dy_t	dz_t	dx_t	dy_t	dz_t	dx_t	dy_t	dz_t
dx_s	0.6	0.6	0.0	0.0	1.1	0.4	0.2	1.1	0.5	0.2
dy_s	5.2	9.3	2.8	4.0	6.5	1.1	3.6	6.5	2.1	3.4
dz_s	8.2	10.7	2.8	3.5	7.3	7.1	4.1	5.3	5.9	5.4
dv_x	0.9	8.0	23.7	31.9	5.6	40.6	60.7	1.9	34.3	68.4
dv_y	7.4	127.9	377.6	509.6	33.0	241.4	360.8	11.6	205.8	410.4
dv_z	11.5	121.6	359.2	484.6	41.1	300.9	449.8	9.8	173.9	346.7
dR	2.8	6.8	0.7	1.1	6.4	2.8	1.2	6.3	3.0	1.3
Total		177	522	704	54	388	580	19	272	542

Table 3. Typical error sources in the ERS-2 SAR images and their effect on positioning in *x, y* and *z* direction (unit: meter)

From Tab.3 it can be found that the maximum positioning errors are caused by the satellite velocity uncertainties whose maximum reach 703.95m, 579.81m and 541.58m in three sites in the *z* direction, respectively. It almost equals to the total errors in that direction, so reducing the orbit errors is critical to the accurate positioning using R-D model. If the other errors are ignored, the required absolute accuracy of orbit determination should be in the order of 19cm to achieve a positioning accuracy of around 25 m predicted by Curlander (Curlander, 1982) using the R-D model. So the precise orbit, which can be provided for free by many organizations, must be

used. The error caused by atmospheric delay is the smallest among all of these errors and its maximum error is only 6.8m in three groups of test. But in Taiwan test site the atmospheric delay error accounts for 33％ of the total error in *x* direction. So it may become remarkable in low latitude area. Moreover, comparing the results of Tibet Zhang autonomous region, Nanjing city and Taiwan province it can be found that the positioning errors using R-D model are nonlinear. They decrease gradually with decreasing latitude as well as increasing longitude. At last, it should be noted that the max. *z* direction error is 704 meter, but it is permitted in the technical

document of RSI Co. Thus, the pixel location using R-D model is effective.

4. CONCLUSIONS

Error models in pixel positioning base on R-D model have been derived for the effect of orbit determination error, earth model error, Doppler center frequency error and atmospheric propagation delay error. By the computing and analysis in three test sites, the following results can be concluded

1. The earth model error can introduce remarkable error, especially in high elevation area as Tibet Zhang autonomous region, thus the exact earth model must be used.

2. In most cases, the zero-Doppler condition can not be satisfied, thus its true-value must be computed.

3. Once the exact earth model and Doppler center frequency true-value have been used, the uncertainty in the orbit determination becomes the dominant factor. If the precise orbit were not adopted, the maximum error of three groups of test is 704 meters which is still permitted in the technical document of RSI Co.. When the precise orbit is used, the maximum error can be improved to 19m. Thus, the pixel location using R-D model is effective.

4. Compared with the orbit error, the atmospheric delay error is smaller, but it also needs to be considered in low latitude areas.

5. The positioning errors are nonlinear over the globe. They decrease gradually with decreasing latitude as well as increasing longitude.

From the equations (1)~(3), we can see that the R-D model can also be used to improve the SAR satellite ephemeris data when accurate reference points, such as radar corner reflectors, exist. Accurate satellite ephemeris data are very important to the many applications of SAR images, such as differential interferometric synthetic aperture radar (DInSAR) technique. Analying on this will be discussed in the future.

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