AN OBJECT-BASED LAND-USE CELLULAR AUTOMATA MODEL TO OVERCOME CELL SIZE AND NEIGHBORHOOD SENSITIVITY

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ABSTRACT:

Cellular automata (CA) are individual-based spatial models increasingly used to simulate the dynamics of natural and human systems and forecast their evolution. Despite their simplicity, they can exhibit extraordinary rich behavior and are remarkably effective at generating realistic simulations of land-use patterns and other spatial structures. However, recent studies have demonstrated that the standard raster-based CA models are sensitive to spatial scale, more specifically to the cell size and neighborhood configuration used for the simulation. To overcome spatial scale dependency, a novel object-based CA model has been developed where space is represented using a vector structure in which the polygons correspond to meaningful geographical entities composing the landscape under study. The proposed object-based CA model allows the geometric transformation of each polygon, expressed as a change of state in part or in totality of its surface, based on the influence of its respective neighbors. In addition, the concept of dynamic neighborhood has been implemented where the neighborhood relationships among objects are defined semantically, that is two objects are neighbors if they are separated by 0, 1 or more objects whose states favor the state transition between them. This flexible neighborhood definition removes any restriction of distance to identify neighborhood relationships among objects, therefore overcoming the neighborhood configuration sensitivity present in the traditional raster CA models. The model was tested to simulate the land-use/land-cover changes in a sub-area of the Elbow river watershed, located in Southwest Alberta, Canada. The results reveal that the object-based CA model generates an adequate evolution of the geographic objects and a spatial configuration of the landscape patches that is more realistic than the one produced by a conventional raster-based CA model. The model also produces land-use patterns that are very similar to the reference maps.

1. INTRODUCTION

Cellular Automata (CA) are dynamic models that can be used to simulate the evolution of a wide variety of natural and human systems. CA were originally conceived by Ulam and Von Neumann in the 1940 with five basic components: 1) a grid tessellation on which the model acts; 2) a collection of cell states; 3) a neighbourhood that influences the state of the central cell; 4) transition rules that determine the dynamics of the system; and 5) discrete time steps (Von Neumann and Burks 1966). These models are capable of reproducing complex global patterns and behaviors through simulating local interactions among individual cells (Wolfram, 1984), and are increasingly used for modelling various spatial phenomena including land-use and land-cover changes (Almeida et al. 2003; Li and Yeh 2002; Ménard and Marceau 2007; Wu 2002), urban growth (Dietzel and Clarke 2006; White and Engelen 1993; White et al. 2000), fire propagation (Berjak and Hearne 2002; Favier et al. 2004), among others.

However, scientists have recently pointed out that the traditional raster-based CA models are sensitive to the modifiable units that are used; more specifically the modelling results may vary according to the cell size and the neighborhood configuration. Jenerette and Wu (2001) have used two different cell resolutions in their CA model to study urban expansion and have shown that it generates significantly different land-use patterns. Chen and Mynett (2003) have investigated the impact of cell size and neighborhood configuration in a raster-based CA prey-predator model and observed that different cell sizes and neighborhood configurations affect both the resulting spatial patterns and the system stability. Jantz and Goetz (2005) have examined the modeling results of a widely used CA based urban model, SLEUTH, in response to different cell sizes and have indicated that the cell size at which the land-use data are represented can impact the quantification of the land-use patterns and the ability of the model to replicate spatial patterns. Ménard and Marceau (2005) and Marceau et al. (2008) have also demonstrated that their raster-based land-use CA is sensitive to different cell resolutions and neighborhood configurations. Similarly, Kocabas and Dragicevic (2006) have shown that both the neighborhood size and configuration have a significant influence on the outcomes of a raster-based CA urban growth model.

One approach to minimize the scale sensitivity is to represent space using an irregular tessellation rather than the traditional regular grid. In the initial work undertaken towards this objective, Voronoi diagrams, Delaunay triangles have been used to construct CA models (Shi and Pang 2000; Flache and Hegselmann 2001; Semboloni 2000), in which space is partitioned into Voronoi polygons or Delaunay triangles, and the neighborhood is defined as the adjacent Voronoi polygons or Delaunay triangles. These CA models are able to depict the spatial interactions between any irregular spatial objects and to simulate their dynamics. Another pioneer irregular CA is the graph CA developed by O'Sullivan (2001a, 2001b) in which space is represented as a planar graph composed of vertexes and edges. Each vertex stands for an object and its neighborhood is defined as a set of vertices linked to it by edges. These studies have demonstrated the usability of the irregular tessellation in CA models. One limitation of these approaches however, is that the polygons are generated automatically and might not correspond to the real-world entities composing the landscape. In addition, the neighborhood definition is rigid and limited since it relies only on topology (White and Engelen 2000).

To improve space representation in CA models, Benenson et al. (2002) applied an entity-based approach to simulate urban residential dynamics, where real-world infrastructure elements are directly described, while the neighborhood still remains defined by Voronoi polygons. Torrens and Benenson (2005) introduced the geographic automata system (GAS) combining characteristics of both CA and multi-agent models, where irregular vector objects are used to represent real-world entities composing an urban system. Stevens et al. (2007) set up their CA model based on irregular cadastral land parcels; the neighborhood is defined by the adjacent parcels, parcels accessible from a road, and parcels within a buffer. These CA models are more flexible and more spatially realistic than the traditional raster-based CA models. However, the geometry of the objects remains invariant, that is, the models do not allow the change of shape and size of the objects, which is an important limitation since such changes are prevalent in the real world.

Following the work initiated by Torrens and Benenson (2005), Hammam et al. (2007) use vector agents (VA) to overcome the limitation of fixed geometry in the GAS model. A vector agent is goal-oriented, adaptable, defined by a Euclidian geometry, and able to change its own geometry while interacting with other agents in its neighborhood using a set of rules. VA allows real-world objects to be naturally defined and the geometry of the objects explicitly controlled, including their location. It has been shown that the spatial patterns simulated by the vector agents are similar to land-use parcels and urban patches as they appear in the real world. However, at this point, the vector agents are predominately driven by geometry and the transition rules do not explicitly capture the driving factors responsible for the dynamic geometric changes.

Recently, a new object-based CA model (VecGCA) has been proposed by Moreno and Marceau (2006, 2007) and Moreno et al. (2008a) which incorporates both of the two key characteristics: a vector representation of real-world entities and their geometrical transformation. In VecGCA, space is represented as a collection of interconnected irregular geographic objects corresponding to real-world entities. The neighborhood is defined dynamically and is specific to each geographic object at each time step. The geographic objects evolve through time according to a transition function that determines their change of shape and area, which depends on the area and number of the neighbors within the neighborhood and their influence on the specific geographic object. An innovative aspect of the VecGCA model is that the procedure allows the change of geometric transformation of objects and the model is independent of cell size and neighborhood configuration, which overcomes the scale sensitivity in the traditional definition of CA.

This paper presents an application of VecGCA to simulate the land-use/land-cover changes in a sub-area of the Elbow river watershed, located in Southwest Alberta, Canada. Its performance was assessed through visual and quantitative analyses of the shape and distribution of the spatial patterns that are generated when compared to the patterns present in the reference land-use maps of the study area.

2. STUDY AREA

The study area corresponds to a sub-region of the Elbow river watershed, located in Southwest Alberta, Canada, which covers approximately 731 km^2. This subregion comprises the land area drained by the Elbow River and its tributaries, excluding the portion corresponding to the Alberta's Rocky Mountains. Three land-use maps generated from Landsat Thematic Mapper images acquired in the summer of 1996, 2001 and 2006 were available for that study. The landscape is fragmented and composed of numerous polygons of small extent. Three predominant land uses were identified in the region: forest, agriculture and urban. An additional land use corresponds to the T'su Tina Nation reserve; however the dynamics inside this reserve was not simulated.

3. LAND USE CHANGE VECGCA MODEL

3.1 Space

Space is a key component in the VecGCA model, which have been modified from the original definition of space in the classical raster-based CA model. In VecGCA, space is defined as a collection of geographical objects that change through time due to the neighbors' influence and according to a transition function that depends on the transition probabilities and the objects' area.

A geographic object is a representation of a real-world entity, for example a city, a lake, an agricultural area, a forested patch, among others. Each geographic object changes shape through time due to its neighbors' influence which produces the change of state of portions of the object that are removed from this object and joined to the corresponding neighbor. The change of state is produced in the closest area to the neighbor that exerts an influence that is higher than a threshold value (λ), which represents the resistance of a geographic object to a change of state.

Space is represented as a collection of patches of different land uses, where each patch corresponds to a polygon of the vector land-use map of the study area.

3.2 Dynamic neighborhood

VecGCA incorporates a new neighborhood definition, called dynamic neighborhood; there is no distance or fixed area that delineates it and it is specific to each geographic object. The neighborhood includes the whole geographic space, and the neighborhood relationships between two objects depend on the properties of each geographic object. Objects A and B are neighbors if they are adjacent or separated by other objects which states are favorable to the change of state from A to B. A $n \times m$ binary matrix describes if a state X is favorable to the transition from the state Y to Z, where n is the number of possible states of a geographic object and m is the number of possible transitions in the model. In this matrix, the 1 values indicate that a state X is favorable to a transition and the 0 values indicate the opposite. The number of intermediate objects between two objects A and B can be 0, 1 or any number. For the Elbow river watershed land-use changes model, this matrix is calculated from the analysis and comparison of two land-use maps of different dates (1996 and 2001) according to the procedure described in Moreno et al. (2008b).

When an object A is neighbor of an object B, A exerts an influence on B that can produce a change of state in a portion or the totality of its surface. The influence value is variable on the object's surface; it increases when the neighbor is closer to the object to reach the maximum value (g_{max}) on the object's border and it decreases inside the object (Equation 1). If g_{max} is higher than λ, then a geometrical transformation procedure is performed.

$$g_{AB} = \begin{cases} 1 - e^{\alpha_{AB}} & \text{if} \quad 0 \le \alpha \le \alpha' \\ e^{-(\alpha_{AB} - \alpha_{AB}')} & \text{if } \alpha > \alpha' \end{cases} \tag{1}$$

where

g_{AB} is the influence of A on B,
α_{AB} is defined in Equation 2 and
α_{AB}' is the value α_{AB} on the B's border.

$$\alpha_{AB} = p^{1/2} \left(\frac{\frac{a_A}{a_B}}{\frac{a_{\max}}{a_{\min}}} + \frac{cb}{b_B} + e^{-d_{\min}} \right) \tag{2}$$

$$0 \le \alpha \le 3$$

where

p is the transition probability from the B's state to A's state,
a_A is the A's area,
a_B is the B's area,
a_{max} is the largest object's area within the whole geographic space,
a_{min} is the smallest object's area,
cb is the common border between A and B,
b_B is the B's perimeter, and
d_{min} is the minimum distance between A and B.

A detailed description of these equations is presented in Moreno et al. (2008b). The transition probabilities are obtained from the comparison of 1996 and 2001 land-use maps and are calculated as the area that changes from the state X at time t to the state Y at $t+1$ divided by the total area that changes from the state X to all other states at $t+1$ (including Y). For a temporal resolution of one year, the transition probabilities were calculated using the exponential method presented by Yeh and Li (2006), where the transition probability P calculated for a time step t is substituted by P^n for a time step T where $T = n*t$.

3.3 Geometric transformation procedure

This procedure performs the change of shape on a geographic object. When a neighbor exerts an influence higher than a threshold value, a buffer around that neighbor is built and the intersection area between this buffer and the geographic object that receives the influence is removed from that object and joined to the corresponding neighbor. The size of the buffer that is built around the neighbors is defined in the transition function.

3.4 Transition function

As the influence of a neighbor on a geographic object decreases inside the geographic object, the transition function is defined as the distance from the geographic object's border to any point inside the object where the influence value is equal to the threshold value (Figure 1). That is, the transition function calculates the value of d_{min} for which the influence value is equal to the threshold value (λ) when α_{AB} is higher than α_{AB}' (Equation 3).

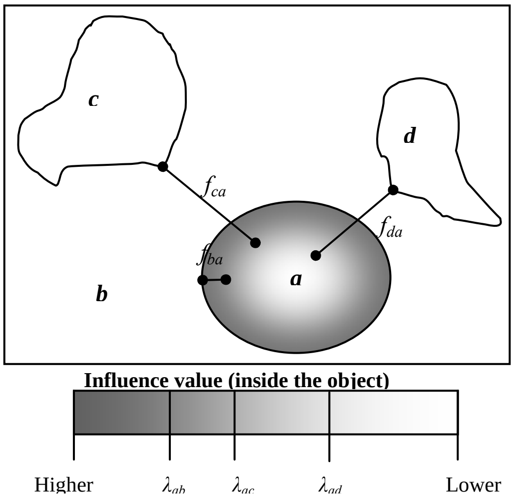

Figure 1: Schematic representation of the distance calculated from the transition function that defines the buffer size to be built around each neighboor to take a portion of a geographical object

$$f_{AB} = -\beta * Ln\left[\frac{1}{p^{1/2}} \left(\alpha_{AB}' - Ln(\lambda) \right) - \frac{\frac{a_A}{a_B}}{\frac{a_{\max}}{a_{\min}}} - \frac{cb}{b_B} \right] \tag{3}$$

where

f_{AB} is the transition function that determines the size of the buffer that is built around A to take a portion of B,
β is a random variable limited between 0 and 1, that represents the influence of other driving factors that are not included in the transition function.

3.5 Model simulations

Simulations were performed from 1996 to 2016, and the results obtained were compared to the 2001 and 2006 land-use/land-cover maps. The temporal resolution of the simulations is one year. Since a pseudo-random number generator was used to implement the random variable (β) in the transition function, five replicates of each simulation were performed for each model and the mean was calculated.

VecGCA was implemented in Java and it uses two additional libraries: OpenMap library (OpenMap 2005) for the handling and display of shape files, and JTS Topology Suite (JTS 2004) for the handling of geometric objects (points, lines, polygons, polylines), buffer construction and geometric operations (intersection, difference, union, etc).

4. RESULTS

The simulation results obtained reveal a good performance of the model independently of the cell size, neighborhood and landscape configuration. They are very similar to the landscape configuration represented on the 2001 land-use map. The proportion of forested land for 2001 corresponds to 44.24 % in comparison to 46.49% of forested land calculated from the 2001 land-use map (Table 1). The proportion of land uses generated by VecGCA varies by less than 3 percentage points when compared to the 2001 land-use map. The results reveal a high correspondence of the landscape generated by VecGCA using a dynamic neighborhood and the landscape present in the 2001 land-use map. A proportion of 98.21, 87.24 and 87.98 % of forested, agricultural and urban areas, respectively, corresponds to the forested, agricultural and urban patches present in the 2001 land-use map (Table 1). Additionally, the spatial autocorrelation represented in the Moran index is very similar for the 2001 simulation outcomes and the 2001 reference land-use map, being 0.12 and 0.15, respectively.

Land uses	Proportion of simulated land uses	Reference Land-use map	Simulated land-uses – reference land-use map	% coincident with the ref. land-use map
Forest	44.24	46.49	-2.25	98.21
Agriculture	25.90	23.35	2.55	87.24
Urban	5.63	5.40	0.23	87.98

Table 1. Proportion of land uses produced by VecGCA for 2001 and percentage of coincidence of the simulation outcomes and the reference land-use maps.

The same analysis performed using the 2006 land-use map (Table 2) leads to similar results which suggests that the model adequately captures the dynamics in the region. When the model was run over an additional 10 years (until 2016) the trends remain the same: the forest and agricultural areas slightly decrease while the urban area increases steadily until 2014 where it seems to reach a plateau (Figure 2).

Land uses	Proportion of simulated land uses	Reference Land-use map	Simulated land-uses – reference land-use map	% coincident with the ref. land-use map
Forest	44.12	45.01	-0.89	95.68
Agriculture	25.39	24.98	0.41	89.47
Urban	6.26	6.18	-0.08	85.48

Table 2. Proportion of land uses produced by VecGCA for 2006 and percentage of coincidence of the simulation outcomes and the reference land-use maps.

Figure 3 presents the simulated maps that were obtained. A visual comparison of these maps with the 2006 reference land-use map reveals the similitude between the spatial patterns generated by VecGCA and those present in the study area.

The results obtained with the implementation of a new dynamic neighborhood in VecGCA demonstrate that the model can adequately represent the dynamics in the study areas. This neighborhood definition removes the neighborhood size sensitivity of the classical raster-based CA models and the previous vector-based CA models because it allows the representation of all possible neighborhood sizes in a unique neighborhood configuration. The neighbors of a geographic object can be adjacent or separated by any distance where the limit is the extent of the geographic space.

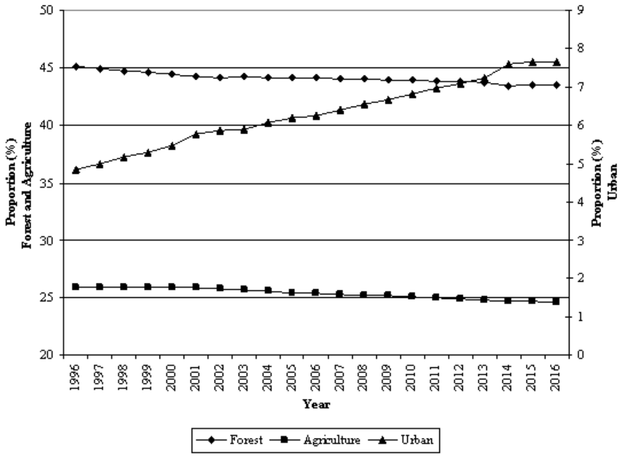

Figure 2. Proportion of Forest, Agriculture and Urban areas for the simulation period 1996-2016.

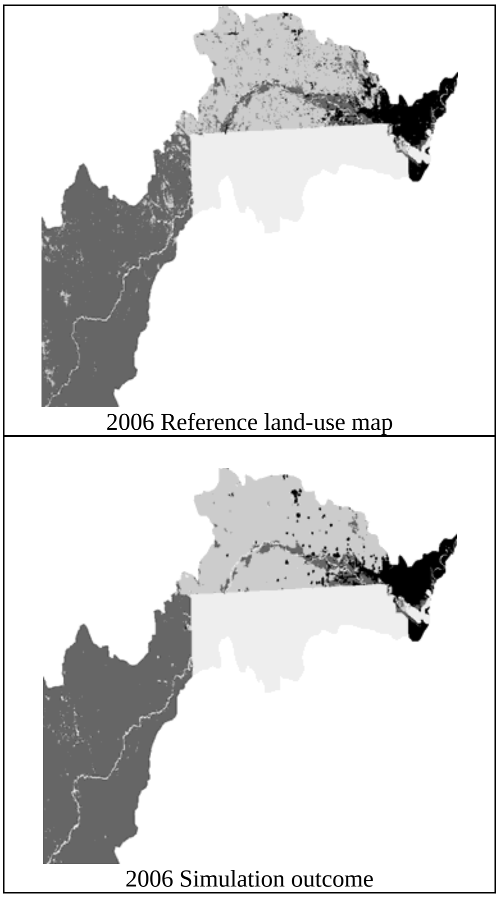

Figure 3. Spatial patterns displayed on the reference land-use map and the simulations performed with VecGCA

VecGCA using a dynamic neighborhood is computationally intensive due to the number of geometric operations executed in the selection of the neighbors in addition to the geometric operations in the change of shape of the objects. For the Elbow river watershed model, the computation time was approximately 60 hours (for ten iterations from 1996 to 2006) for each replication. However, using VecGCA with a dynamic neighborhood eliminates the computation time associated to the sensitivity analysis that should be conducted to determine the best combination of cell size and neighborhood configuration in a raster-based CA model, which is a considerable advantage.

5. CONCLUSION

VecGCA is a new object-based geographical cellular automata model presented as a solution to overcome the limitations of raster-based CA models and previous vector-based models. The space definition as a collection of real-world objects (polygons) with proper behavior that evolves through time overcomes the cell size sensitivity.

In addition, the new dynamic neighborhood removes the neighborhood configuration sensitivity, because the neighborhood relationships among objects are described semantically and are not associated to a fixed distance. In a dynamic neighborhood, an object *A* is neighbor of another object *B* if they are separated by 0,1 or more objects which states are favorable to the change of state from *A* to *B*. The principal advantage of this new neighborhood definition is that it is independent of a fixed influence zone and it uses the whole geographic space to evaluate which geographic objects exert an influence on others to generate a geometric transformation or change of shape. The dynamic neighborhood encompasses all possible neighborhood sizes in a unique neighborhood configuration; there are not fixed areas that limit the neighborhood relationships between objects.

The results reveal that VecGCA produces spatial patterns similar to the reference land-use maps, which suggests that it represents adequately the dynamics in the study area. Sensitivity analyses to determine the most appropriate cell size or neighborhood configuration are not required anymore.

A random factor was included in the transition function to represent external driving forces that participate in the dynamics of the study area, but the model was mainly based on transition probabilities. On-going researches aim at developing a more complex land-use change model that includes the significant driving factors that participate in the land-use dynamics of the study area.

The new object-based model VecGCA is a generic powerful tool to simulate land-use/land-cover changes or other spatio-temporal phenomena that implies geometric transformations of objects. With the inclusion of a dynamic neighborhood, VecGCA becomes independent of the cell size, the neighborhood configuration and the landscape configuration and ensures a more realistic representation of geographic space and evolution of the objects composing it.

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