Production Planning and Control

Professor JIANG Zhibin
Dr. GENG Na

Department of Industrial Engineering & Logistics Management
Shanghai Jiao Tong University

Chapter 3 Aggregate Planning

Dr Na GENG

Department of Industrial Engineering & Management

Shanghai Jiao Tong University

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Contents

A Prototype Problem

Example 3.2

Densepack is to plan workforce and production level for six-month period Jan. to June. The firm produces a line of disk drives for mainframe computers. Forecast demand over the next six months for a particular line of drives in a plant are 1,280, 640, 900, 1,200, 2,000 and 1,400. There are currently (end of Dec.) 300 workers employed in the plant. Ending inventory in Dec. is expected to be 500 units, and the firm would like to have 600 units on hand at the end of June.

A Prototype Problem

Month	Predicated Demand	Net Predicated Demand	Net Cumulative Demand
Jan.	1,280	780(1280-500)	780
Feb.	640	640	1,420
March	900	900	2,320
April	1,200	1,200	3,520
May	2,000	2,000	5,520
June	1,400	2,000(1400+600)	7,520

A Prototype Problem

If the shortage is not permitted, the cumulative production must be at least as great as cumulative net demand each period-Feasible AP

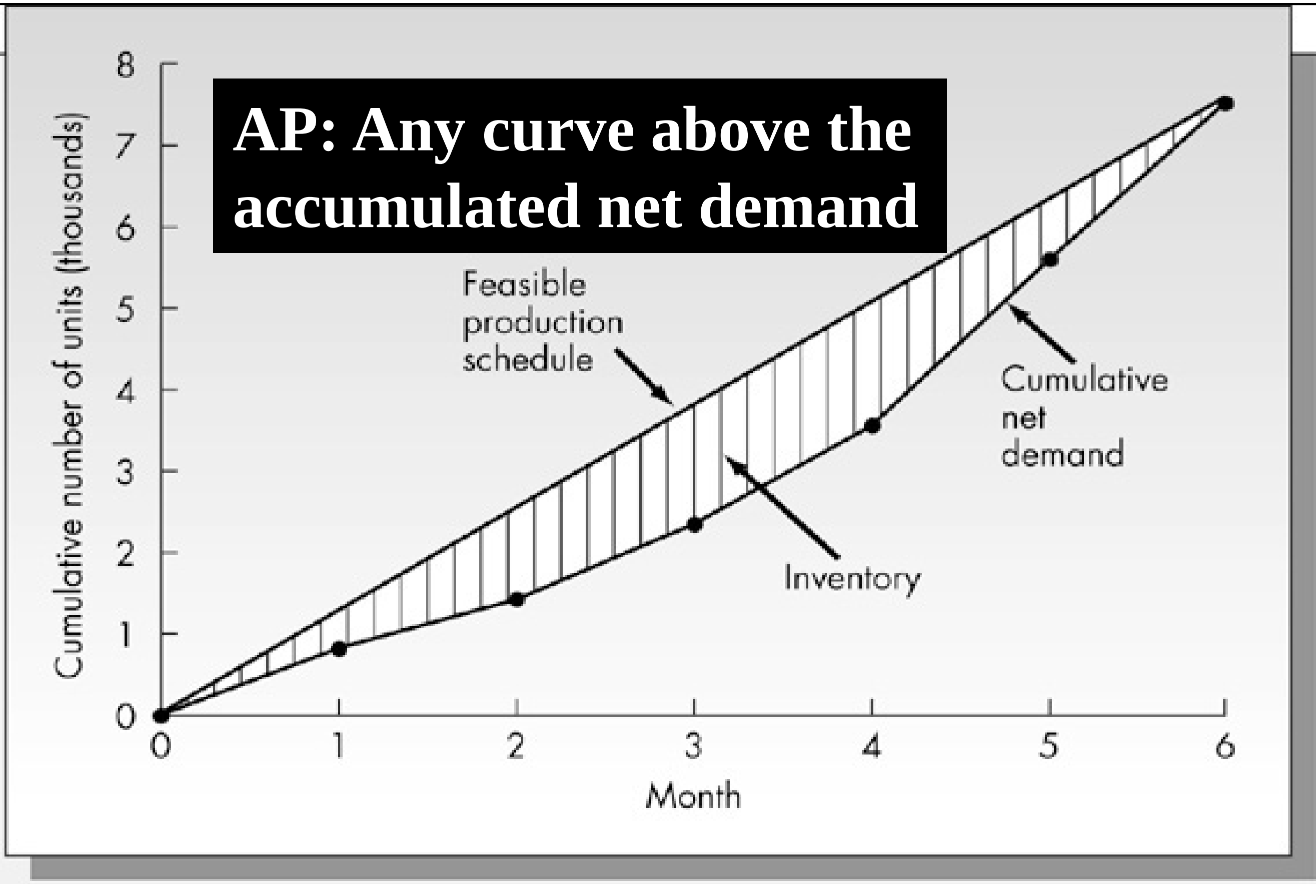

Fig. 3-4 A Feasible Aggregate Plan for Densepack

A Prototype Problem

- **How to make cost trade-offs of various production plans?**

Only consider three costs:

✓C_H=Cost of hiring one worker=$500;

✓C_F=Cost of firing one worker=$1,000;

✓C_I=Cost of holding one unit of inventory for one month=$80

A Prototype Problem

•Translate aggregate production in units to workforce levels:

- ✓ Use a day as an indivisible unit of measure (since not all month have equal number of working days) and define:
- ✓ K=Number of aggregate units produced by one worker in one day.
- ✓ A known fact: over 22 working days, with the workforce constant at 76 workers, the firm produced 245 disk drives.
- ✓ Average production rate=245/22=11.1364 disk drives per day;
- ✓ One worker produced an average of 11.1364/76=0.14653 drive in one day. K=0.14653.

A Prototype Problem

Two alternative plans for managing workforce:

- Plan 1 is to change workforce each month in order to produce enough units to most closely match the demand pattern-zero inventory plan;
- Plan 2 is to maintain the minimum constant workforce necessary to satisfy the net demand-constant workforce plan;

A Prototype Problem

P1: Zero Inventory Plan (Chase Strategy) – minimize inv. level.

Table 3-1 Initial Calculation for Zero Inv. Plan for Denspack

A	B	C		
Month	No. of Working Days	No. of Units Produced per Worker (B×K)		
Jan.	20	2.931		
Feb.	24	3.517		
March	18	2.638		
April	26	3.810		
May	22	3.224		
June	15	2.198		

A Prototype Problem

P1: Zero Inventory Plan (Chase Strategy) – minimize inv. level.

Table 3-1 Initial Calculation for Zero Inv. Plan for Denspack

A	B	C	D	
Month	No. of Working Days	No. of Units Produced per Worker (B×K)	Forecast Net Demand	
Jan.	20	2.931	780	
Feb.	24	3.517	640	
March	18	2.638	900	
April	26	3.810	1,200	
May	22	3.224	2,000	
June	15	2.198	2,000	

A Prototype Problem

P1: Zero Inventory Plan (Chase Strategy) – minimize inv. level.

Table 3-1 Initial Calculation for Zero Inv. Plan for Denspack

A	B	C	D	E
Month	No. of Working Days	No. of Units Produced per Worker (B×K)	Forecast Net Demand	Minimum No. of Worker required (D/C rounded up)
Jan.	20	2.931	780	267
Feb.	24	3.517	640	182
March	18	2.638	900	342
April	26	3.810	1,200	315
May	22	3.224	2,000	621
June	15	2.198	2,000	910

A Prototype Problem

- The number of workers employed at the end of Dec. is 300;
- Hiring and firing workers each month to match forecast demand.

Table 3-2 Zero Inv. Aggregate Plan for Densepack

A	B	C	D	E	F	G	H	I
Month	No. of Workers	No. Hired	No. Fired					
Jan.	**267**		**33**					
Feb.	**182**		**85**					
March	**342**	**160**						
April	**315**		**27**					
May	**621**	**306**						
June	**910**	**289**						
Total		**755**	**145**					

A Prototype Problem

- The number of workers employed at the end of Dec. is 300;
- Hiring and firing workers each month to match forecast demand.

Table 3-2 Zero Inv. Aggregate Plan for Densepack

A	B	C	D	E	F	G	H	I
Month	No. of Workers	No. Hired	No. Fired	No. of Units per Worker	No. of Units Produced (B×E)	Cumulative Production		
Jan.	**267**		**33**	**2.931**	**783**	**783**		
Feb.	**182**		**85**	**3.517**	**640**	**1,423**		
March	**342**	**160**		**2.638**	**902**	**2,325**		
April	**315**		**27**	**3.810**	**1,200**	**3,525**		
May	**621**	**306**		**3.224**	**2,002**	**5,527**		
June	**910**	**289**		**2.198**	**2,000**	**7,527**		
Total		**755**	**145**					

A Prototype Problem

- **The number of workers employed at the end of Dec. is 300;**
- **Hiring and firing workers each month to match forecast demand.**

Table 3-2 Zero Inv. Aggregate Plan for Densepack

A	B	C	D	E	F	G	H	I
Month	No. of Workers	No. Hired	No. Fired	No. of Units per Worker	No. of Units Produced (B×E)	Cumulative Production	Cumulative Demand	Ending Inv. (G-H)
Jan.	**267**		**33**	**2.931**	**783**	**783**	**780**	**3**
Feb.	**182**		**85**	**3.517**	**640**	**1,423**	**1,420**	**3**
March	**342**	**160**		**2.638**	**902**	**2,325**	**2,320**	**5**
April	**315**		**27**	**3.810**	**1,200**	**3,525**	**3,520**	**5**
May	**621**	**306**		**3.224**	**2,002**	**5,527**	**5,520**	**7**
June	**910**	**289**		**2.198**	**2,000**	**7,527**	**7,520**	**7**
Total		**755**	**145**					**30**

A Prototype Problem

- The total cost of this production plan is obtained by multiplying the totals at the bottom of Table 3-2 by corresponding unit cost:

  755×500+145 ×1000+30 ×80=$524,900;

- In addition, the cost of holding for the ending inventory of 600 units, which was considered as the demand for June, should be included in holding cost: 600 ×80=$48,000
- The total cost= $524,900+$48,000=$572,900.
- Note: the initial inventory of 500 units does not enter into the calculation because it will be netted out during the month January.

A Prototype Problem

- It is possible to achieve zero inventory at the end of each planning period ?

No! Since it is impossible to have a fractional number of workers.

- It is possible that ending inventory in one or more period could build up to a point where the size of the workforce may be reduced by one or more workers.

A Prototype Problem

P2 Evaluation of the Constant Workforce Plan-to eliminate completely the need for hiring and firing during the planning horizon.

- In order not incur the shortage in any period, compute the minimum workforce required for every month in the planning horizon.

✓ For January, the net cumulative demand is 780 and units produced per worker is 2.931, thus the minimum workforce is 267(780/2.931) in Jan;

Units produced per worker in Jan. and Feb. combined = 2.931+3.517 = 6.448, and the cumulative demand is 1,420, then the minimum workforce is 221(1420/6.448) to cover both Jan. and Feb.

✓ Go on computing in the same way.

A Prototype Problem

Table 3-3 Computation of the Minimum Workforce Required by Denspack

A	B	C	D
Month	**Cumulative Net Demand**	**Cumulative No. of Units Produced per Worker**	**Ratio B/C (Rounded up)**
Jan.	**780**	**2.931**	**267**
Feb.	**1,420**	**6.448**	**221**
March	**2,320**	**9.086**	**256**
April	**3,520**	**12.896**	**273**
May	**5,520**	**16.120**	**343**
June	**7,520**	**18.318**	**411**

The minimum number of workers required for entire six-month planning horizon is 411, requiring hiring 111 new workers at the beginning of Jan.

A Prototype Problem

Table 3-4 Inventory Level for Constant Workforce Schedule

A	B	C	D	E	F
Month	No. of Units Produced per Worker	Monthly Production (B×411)	Cumulative Production	Cumulative Net Demand	Ending Inventory (D-E)
Jan.	2.931	1,205	1,205	780	425
Feb.	3.517	1,445	2,650	1,420	1,230
March	2.638	1,084	3,734	2,320	1,414
April	3.810	1,566	5,300	3,520	1,780
May	3.224	1,325	6,625	5,520	1,105
June	2.198	903	7,528	7,520	8
Total					5,962+600

- The total cost is (5,962+600)×80+111 ×500=580,460>569,540 for P1;
- P2 is preferred because it has no large difference from P1 in cost, but keeps workforce stable.

A Prototype Problem

Mixed Strategy and Additional Constraints

- The zero inventory plan and the constant workforce strategies are to target one objective;
- Combining the two plans may results in dramatically lower costs;
- Figure 3-4 shows the workforce strategy line-a fixed rate).

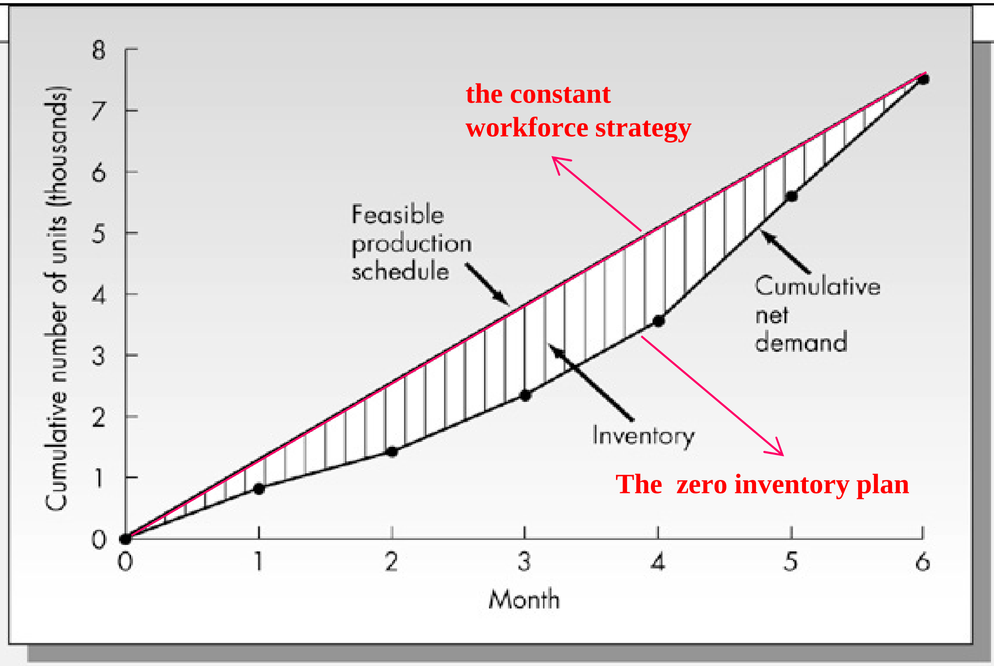

Fig. 3-4 A Feasible Aggregate Plan

A Prototype Problem

Mixed Strategy and Additional Constraints

Suppose that we may use two production rates (2 straight lines):

- Make net inventory at the end of April to be zero (P1) by producing enough in each of the four months Jan. through April to meet the cumulative net demand each month: produce 3,520/4=880 units in each of the first four months;
- The May and June production is then set to 2,000, exactly matching the net demand in these months.

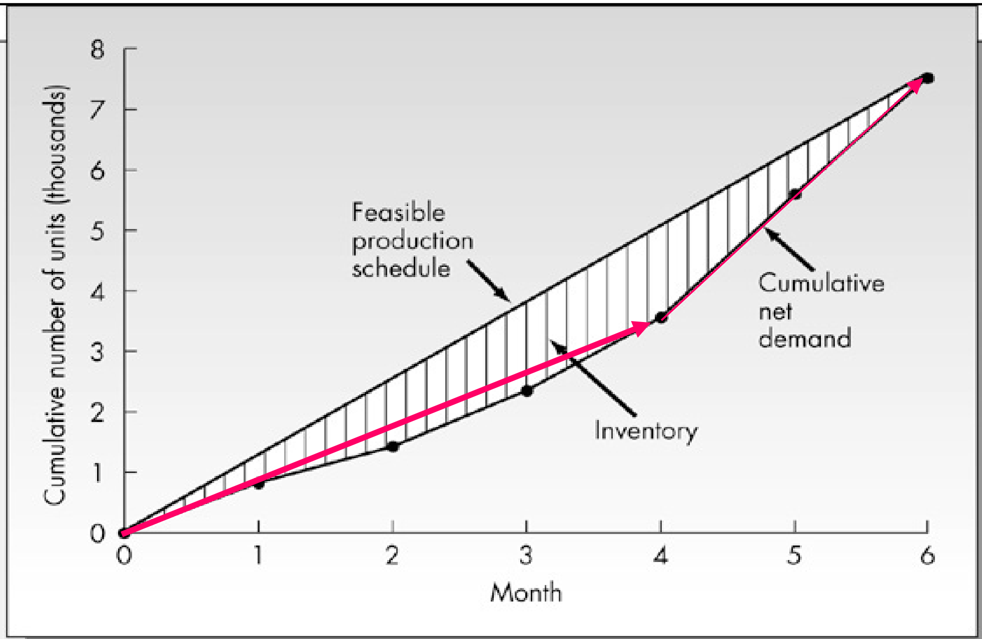

Fig. 3-4 A Feasible Aggregate Plan

The two lines are above the cumulative net demand, the plan is feasible

A Prototype Problem

Month	Cumulative Net Demand	Cumulative Production
Jan.	780	880
Feb.	1,420	1,760
March	2,320	2,460
April	3,520	3,520
May	5,520	5,520
June	7,520	7,520

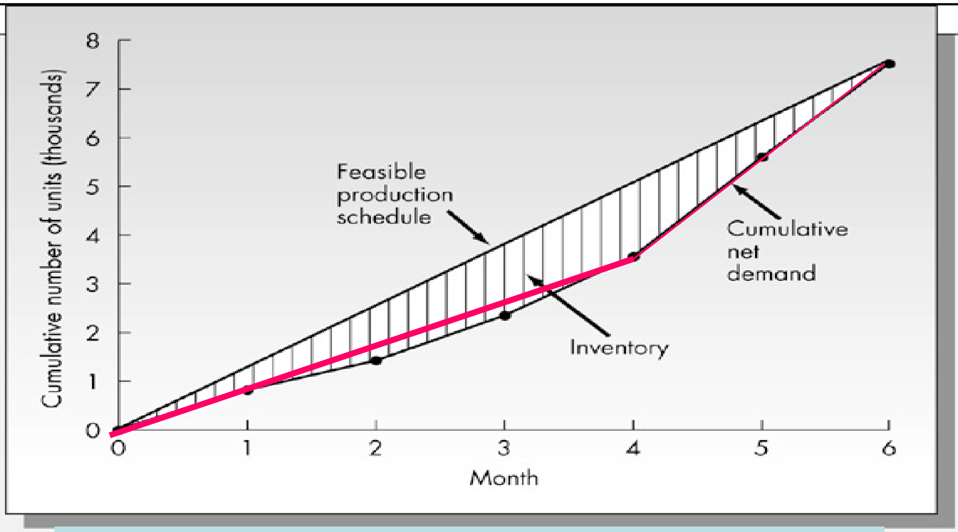

Fig. 3-4 A Feasible Aggregate Plan

A Prototype Problem

The graphical solution for additional constraints.

•Capacity limitation: the production capacity of the plan is only 1,800 units per month

•One feasible solution: produce 980 in each of the first four months and 1,800 in each of the last two months. (Slopes of the two segmental lines ≤1800)

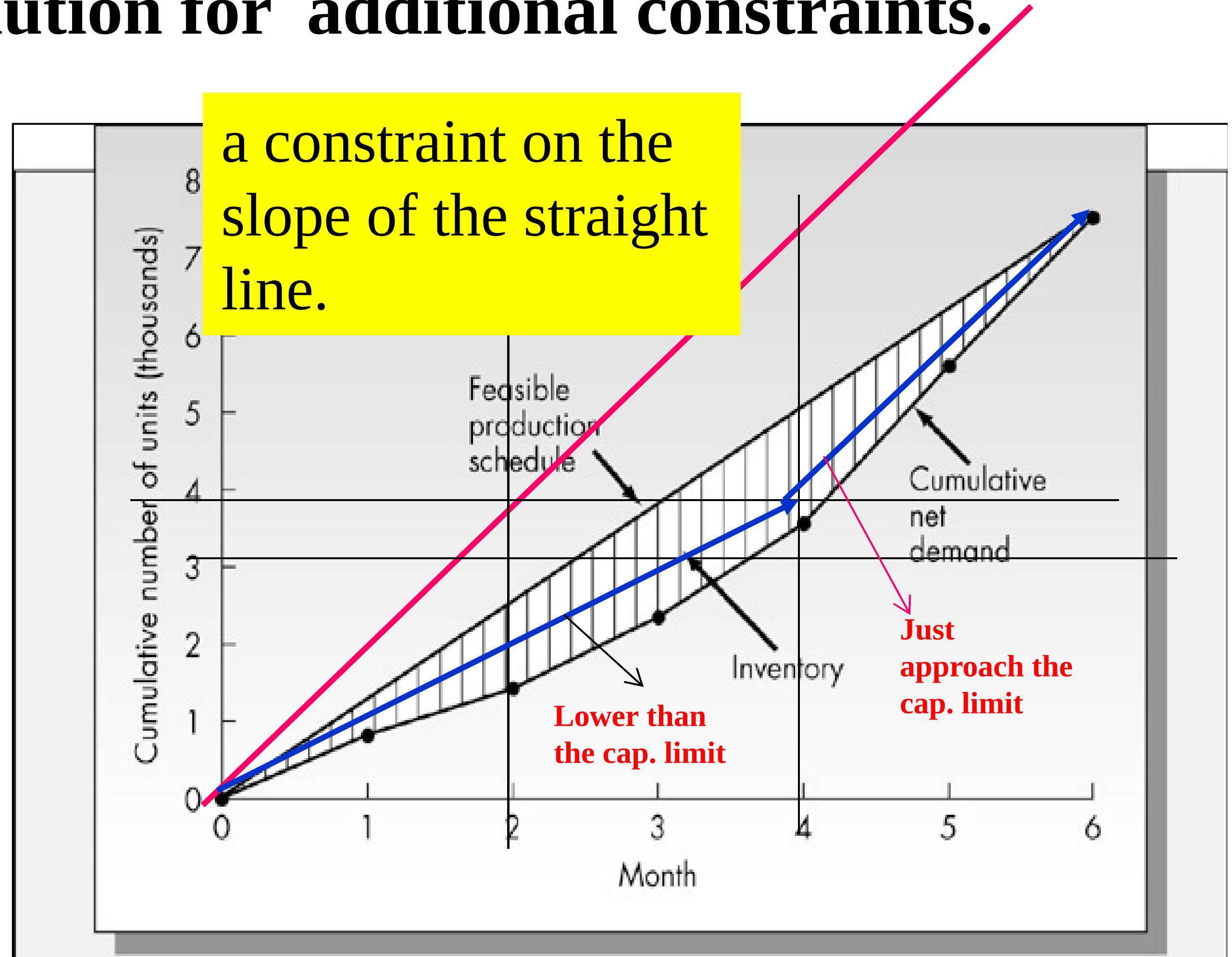

Fig. 3-4 A Feasible Aggregate Plan

Chapter 3 Aggregate Planning

Contents

Aggregate Planning by Linear Programming

Linear Programming (LP) is used to determine values of n nonnegative variables to maximize or minimize a linear function of these variables that is m linear constraints of these variables.

Cost Parameters

C_H=Cost of hiring one worker;

C_F= Cost of firing one worker;

C_I= Cost of holding one unit of stock for one period;

C_R= Cost of producing one unit product on regular time;

C_O= Incremental cost of one unit on overtime;

C_U= Idle cost per unit of production;

C_S= Cost of subcontract one unit of production;

n_t=Number production days in period *t*;

K=Number of aggregate units produced by one worker in one day;

I_0=Initial inventory on hand at the start of the planning horizon;

W_0=Initial workforce at the start of the planning horizon;

D_t=Forecast of demand in period *t*;

Aggregate Planning by Linear Programming

Problem Variables:

- W_t=Workforce level in period t;
- P_t=Production level in period t;
- I_t=Inventory level in period t;
- H_t=Number of workers hired in period t;
- F_t=Number of workers fired in period t;
- O_t=Overtime production in units;
- U_t=Worker idle time in units (underutilized time);
- S_t=Number of units subcontracted from outside;

•If $P_t > Kn_tW_t$: the number of units produced on overtime : $O_t=P_t-Kn_tW_t$;

•If $P_t < Kn_tW_t$: the idle time is measured in units of production rather than in time, $U_t= Kn_tW_t - P_t$;

Aggregate Planning by Linear Programming

Constraints-Three sets of constraints to ensure conservation of labor and that of units

1. Conservation of workforce constraints

   $W_t = W_{t-1} + H_t - F_t$; for $1 \le t \le T$

2. Conservation of units constraints

   $I_t = I_{t-1} + P_t + S_t - D_t$; for $1 \le t \le T$

3. Conservation of relating production level to workforce levels

   $P_t = Kn_t W_t + O_t - U_t$; for $1 \le t \le T$

(1–3: 3T constraints)

4. Others

- Non negative constraints;
- Given I_0, I_T, and W_0.

Aggregate Planning by Linear Programming

Objective function-to choose variables W_t, P_t, I_t, H_t, F_t,O_t, U_t and S_t (total 8T) to

$$Min \sum_{t=1}^{T} (c_H H_I + c_F H_F + c_I I_t + c_R P_t + c_O O_t + c_U U_t + c_S S_t)$$

Subject to

- the above 3T constraints,
- nonnegative constraint: W_t, P_t, I_t, H_t, F_t, O_t, U_t and $S_t \geq 0$, and
- I_0, I_T, and W_0.

Aggregate Planning by Linear Programming

Rounding the Variables

- Some variables such as I_t, W_t, F_t, H_t should be integers;
- May calculate by integer linear programming, however, may be too complex;
- Results of LP should be rounded up- by Conservative approach
  - ✓ Round W_t to the next larger integer, and then calculate H_t, F_t, and P_t;
  - ✓ Always feasible solution, but rarely optimized;

Additional constraints

- O_tU_t=0-either one is zero in case that there are both overtime an idle production in the same period ;
- H_tF_t=0- either in case of hiring and firing workers in the same period
- Both the two constraints are linear;

Aggregate Planning by Linear Programming

Extensions

- Account for uncertainty in demand by minimum buffer inventory B_t: $I_t \geq B_t$, for $1 \leq t \leq T$, where B_t should be specified in advance;
- Capacity constraints on amount of production: $P_t \leq C_t$;
- In some cases, it may be desirable to allow demand exceed the capacity. To treat the backlogging of excess demand, the inventory needs to be expressed in terms of two different non-negative variables I_t^+ and I_t^-, such that $I_t = I_t^+ - I_t^-$, and holding cost is charged against I_t^+, while the penalty cost for back orders against I_t^-.

Solving AP Problems by LP: An Example (3.2)

Since no subcontracting, overtime, or idle time allowed, and the cost coefficients are constant with respect to time, the objective function is simplified as

$$Min\ (500\sum_{t=1}^{6} H_t + 1000\sum_{t=1}^{6} F_t + 80\sum_{t=1}^{6} I_t)$$

$W_1 - W_0 - H_1 + F_1 = 0;$	$P_1 - I_1 + I_0 = 1{,}280;$	$P_1 - 2.931W_1 = 0;$
$W_2 - W_1 - H_2 + F_2 = 0;$	$P_2 - I_2 + I_1 = 640;$	$P_2 - 3.517W_2 = 0;$
$W_3 - W_2 - H_3 + F_3 = 0;$	$P_3 - I_3 + I_2 = 900;$	$P_3 - 2.638W_3 = 0;$
$W_4 - W_3 - H_4 + F_4 = 0;$	$P_4 - I_4 + I_3 = 1{,}200;$	$P_4 - 3.810W_4 = 0;$
$W_5 - W_4 - H_5 + F_5 = 0;$	$P_5 - I_5 + I_3 = 2{,}000;$	$P_5 - 3.224W_5 = 0;$
$W_6 - W_5 - H_6 + F_6 = 0;$	$P_6 - I_6 + I_5 = 1{,}400;$	$P_6 - 2.198W_6 = 0;$

W_i, P_i, I_i, F_i, H_i (i=1-6) ≥ 0;

$W_0 = 300$, $I_0 = 500$, $I_6 = 600$

Solving AP Problems by LP: An Example (3.2)

- Solved by LINGO system;
- The value of objective function is $379,320.90, considerably less than that obtained by either P_1 and P_2, since this value is obtained by fractional values of variables .
- Rounded up result: the total cost=465 × 500+1,000 ×27+(900+600) ×80=$379,500

A	B	C	D	E	F	G	H	I
Month	No. of Worker s	No. Hired	No. Fired	No. of Units per Worker	No. of Units Produced (B×E)	Cumulativ e Production	Cumulativ e Demand	Endin g Inv. (G-H)
Jan.	**273**		**27**	**2.931**	**800**	**800**	**780**	**20**
Feb.	**273**			**3.517**	**960**	**1,760**	**1,420**	**340**
March	**273**			**2.638**	**720**	**2,480**	**2,320**	**160**
April	**273**			**3.810**	**1,040**	**3,520**	**3,520**	**0**
May	**738**	**465**		**3.224**	**2,379**	**5,899**	**5,520**	**379**
June	**738**			**2.198**	**1,622**	**7,521**	**7,520**	**1**
Total		**465**	**27**					**900**

Summary

- AP: translate forecasted demand into the blueprint of workforce level and production level
- Cost considered in AP: Smoothing cost, holding cost, backlog cost, regular time cost, overtime and subcontract cost, underutilization cost
- Solutions: Chase-up strategy, constant workforce level, and LP

Homework for Chapter 3

- P140 Q14
- P141 Q15, 16

The End!