



# Nanosecond zero-order pulsed Bessel beam generated from unstable resonator based on an axicon

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## ABSTRACT

We present what we believe the first experimental observation of a nanosecond zero order pulsed Bessel beam generated from an unstable resonator based on an axicon. The field distribution at the output coupler is simulated by means of Fox–Li iterative computation. It is shown that the experimental result is consistent with the numerical simulation. The application field and forecast of the zero-order Bessel beams are very wide, such as precision cutting and micro-operation experiment. Our experiments offer a new way to output zero-order Bessel beams directly.

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## 1. Introduction

The high-order Bessel beams (BBs), having zero intensity on axis in the maximum non-diffracting distance, great radial intensity gradient and small size of the central dark spot, can be used in atom guiding [1,2]. In contrast to high-order BBs, the zero-order BBs, has high intensity on axis and a bright core transversal section rather than a dark spot. So it can be used in laser precision processing micro-operation [3,4]. The Bessel–Gauss beams (BGBs) are a kind of Bessel beams modulated in amplitude by a Gaussian function. The generation of BBs and BGBs has attracted much attention, and many different experiments have been demonstrated in the recent 20 years [5–9]. These methods can be split roughly into intra-cavity and extra-cavity generation. Extra-cavity generation means illuminating a special optic element with Gaussian beams to generate BBs or BGBs. For instance, using a diffractive axicon, our group has shaped the nanosecond Gauss beams generated from a Nd:YAG laser to a high stable pulsed BGBs [10]. Intra-cavity generation means the BGBs are generated directly from the various resonators [7,8,11,12]. In last year, our group obtained nanosecond pulsed Bessel–Gauss beams from a Nd:YAG axicon-based stable resonator [13]. In contrast to extra-cavity generation, intra-cavity generation outputs BGBs directly from the resonator without external optical elements. Furthermore, another benefit of these schemes is the possibility of realizing intra-cavity frequency conversion of beams.

In this paper, we present a Q-switched Nd:YAG laser with unstable resonator based on axicon. It is capable of supporting approximate BBs. An unstable resonator possesses higher transverse-mode discrimination in favor of the fundamental mode than a stable resonator, and exhibits more similar to ideal BBs than stable resonator which generate BGBs [14]. The characteristic of the unstable resonator is that the paraxial rays in the cavity must escape from the cavity after a limited times round-trip; the geometrical spreading loss is high, and the attainment of a large fundamental mode volume and good spatial mode selection at high Fresnel number. In another word, it can produce output beams of high-power and single-mode beams. Although the theory of axicon based unstable resonator has been reported in a few papers [14–16], the experimental results have not yet been achieved. In this paper, we report the pulsed zero-order BBs generated from the axicon based unstable resonator directly for the first time. The pulse duration about 5 ns (FWHM) is measured, and three-dimensional intensity distribution pattern of the pulses is captured. We show that the experimental results fit well with the numerical simulation.

## 2. Theoretical analysis and numerical simulation

### 2.1. Zero-order Bessel beams

The theory of BBs and BGBs has been discussed in detail in [17–20]. Stable Bessel resonator supports BGBs, while the unstable Bessel resonator approximate supports BBs. In order to compare the unstable resonator mode-field with an ideal zero-order BBs, we briefly review ideal zero-order BBs in this section.

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It is well known that the zero-order Bessel function of the first kind is one solution of the Helmholtz wave equation

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) E(r, t) = 0 \tag{1}$$

The light field of zero-order BBs can be expressed as

$$E(\rho, \varphi, z, t) = \exp[i(k_\beta z - \omega t)] J_0(k_\alpha \rho) \tag{2}$$

where  $J_0$  is the zero-order Bessel function of the first kind,  $k_\alpha$  and  $k_\beta$  are the radial wave vector and longitudinal wave vector, and  $k_\alpha^2 + k_\beta^2 = (2\pi/\lambda)^2$ , and  $\rho, z$ , and  $\varphi$  are the radial, longitudinal, and azimuthal co-ordinates, respectively. From Eq. (2), we can take a conclusion that the intensity pattern in a transverse plane is unaltered by propagating in free space. Light intensity is taken as proportional to

$$U(\rho, \varphi, z, t) = E(\rho, \varphi, z, t) E^*(\rho, \varphi, z, t) \tag{3}$$

Then the last expression of intensity can be written in terms of Eq. (2), the complex amplitude  $E$ , defined immediately following Eq. (3), as

$$U(\rho, \varphi, z, t) = |J_0(k_\alpha \rho)|^2 \tag{4}$$

### 2.2. Resonant modes

An unstable Bessel resonator with an axicon is shown in Fig. 1. The resonator consists of a thin refractive axicon  $M_1$  with index  $n$  and wedge angle  $\alpha$ , and a convex spherical mirror  $M_2$  with radius of curvature  $R$  that is placed a distance  $L$  from the axicon and serves as the output coupler. The flat surface of axicon is coated with high reflective film and the spherical surface of convex mirror is coated with partial reflective film for the oscillation wavelength. Where  $a$  is the beams radius limited by the axicon size or aperture size,  $r_1$  and  $r_2$  are the radial co-ordinates in the axicon and convex mirror plane, and  $\theta$  is the refraction angle given by

$$\theta = (n-1)\alpha \tag{5}$$

And  $L$  is the cavity length chosen by [21]

$$L = \frac{a}{2(n-1)\alpha} \tag{6}$$

In order to study the mode characteristic of laser in an unstable resonator based on an axicon, a finite element method is presented to numerically calculate the laser mode. If the optical fields at the axicon and the convex mirror are written as  $U_1(r_1, \varphi_1)$  and  $U_2(r_2, \varphi_2)$ , where  $(r_1, \varphi_1)$  and  $(r_2, \varphi_2)$  are radial co-ordinates on the axicon  $M_1$  and convex spherical mirror  $M_2$ , respectively. Based

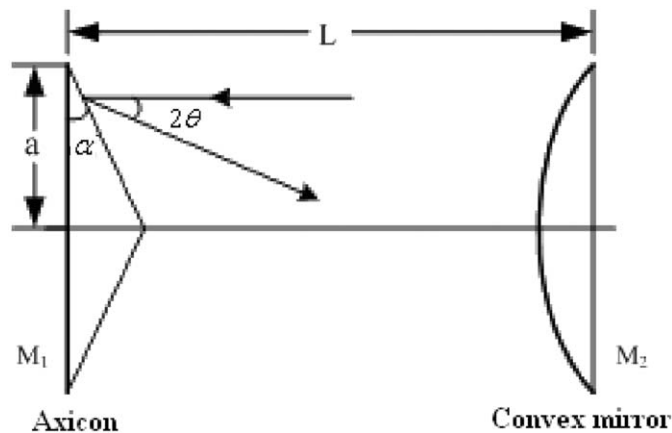


Fig. 1. Scheme of unstable Bessel resonator based on axicon. The axicon is total reflective mirror and the convex mirror is output coupling mirror.

on the Kirchoff diffraction integral theory, the iterative relation between  $U_1(r_1, \varphi_1)$  and  $U_2(r_2, \varphi_2)$  can be written as

$$U_{2,m}(r_2, \varphi_2) = \left(\frac{-i}{\lambda L}\right) \exp(ikL) \iint_{S_1} T_1(r_1) U_{1,m-1}(r_1, \varphi_1) \times \exp\left\{\frac{ik}{2L}[r_1^2 + r_2^2 - 2r_1 r_2 \cos(\varphi_1 - \varphi_2)]\right\} r_1 dr_1 d\varphi_1 \tag{7}$$

and

$$U_{1,m}(r_1, \varphi_1) = \left(\frac{-i}{\lambda L}\right) \exp(ikL) \iint_{S_1} T_2(r_2) U_{2,m}(r_2, \varphi_2) \times \exp\left\{\frac{ik}{2L}[r_1^2 + r_2^2 - 2r_1 r_2 \cos(\varphi_2 - \varphi_1)]\right\} r_2 dr_2 d\varphi_2 \tag{8}$$

where  $m=1,2,3\dots$  is the iterative times;  $T_1(r_1)$  and  $T_2(r_2)$  are the transmittance function of the double axicon and the convex output mirror given by

$$T_1(r_1) = \exp(-i2kr_1\theta) \tag{9}$$

$$T_2(r_2) = \exp(-ikr_2^2/R) \tag{10}$$

Firstly, we assume that the initial state of the optical field on the axicon is plane wave. The other parameters for the unstable resonator: aperture size of the axicon  $a=2$  mm,  $n=1.458$ , wedge angle  $\alpha=0.5^\circ$ , radius of curvature of convex mirror  $R=-2000$  mm, and wavelength of Nd:YAG  $\lambda=1.064$   $\mu\text{m}$ . From expressions (5) and (6), we get refraction angle  $\theta=0.229^\circ$  and cavity length  $L=250$  mm. Typical intensity profiles of unstable resonator mode-field on the output coupling convex mirror after simulation by using Fox–Li iterative computation are shown in Fig. 2.

Secondly, starting from a random noise field, by Fox–Li iterative computation, the field of this resonator also converges to approximate BBs. In order to compare the unstable resonator mode-field with an ideal zero-order BBs. The intensity structure for an ideal zero-order BBs is also presented in Fig. 3 in terms of Eq. (4).

Comparing radial intensity profiles of the unstable resonator based on an axicon mode-field with ideal zero-order BBs, we can obtain a conclusion that laser beams generated from the unstable resonator based on an axicon directly is similar to the ideal zero-order BBs. They have a high intensity spot in the centre of light beams, dark and bright rings circumscribe the central spot alternately, and intensity of bright rings decrease gradually from centre outward. The only difference is that intensity of bright rings of the zero-order BBs from the unstable resonator attenuates much quickly than that of ideal zero-order BBs. Therefore, a few numbers of bright rings is achieved. The main reason is that the laser beams is modulated uniformly by a near-top-hat function [14], unlike the power being shared between the rings of the ideal zero-order BBs. The energy of output beams at unstable resonator is more concentrated.

### 3. Description of the cavity

The unstable resonator generating zero-order BBs is a simple and compact linear cavity. Fig. 4 shows the schematic of the Nd:YAG laser designed for this experiment, where a 80 mm long and 5 mm diameter Nd:YAG rod is pumped with one flash lamp which worked with 0.4–0.6 kV A.C electric discharge in a pulsed state; the flat surface of the axicon has a high-reflectivity coating at the laser wavelength of 1.064  $\mu\text{m}$  and serves as the total

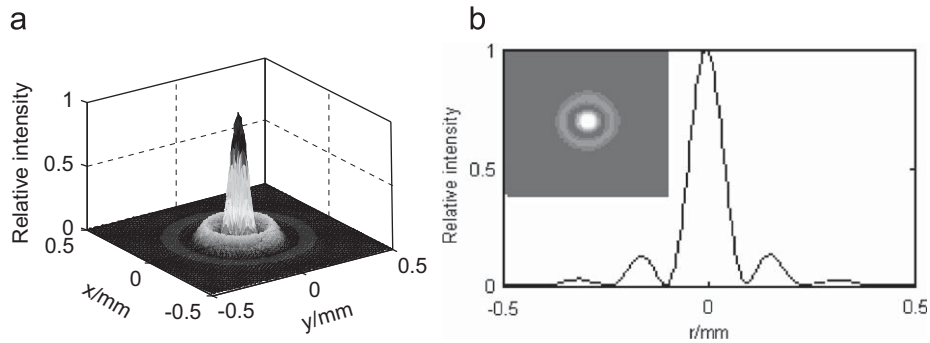


Fig. 2. Simulated intensity profiles of unstable resonator mode-field on the output coupling mirror: (a) three-dimensional distribution and (b) beam pattern and radial distribution.

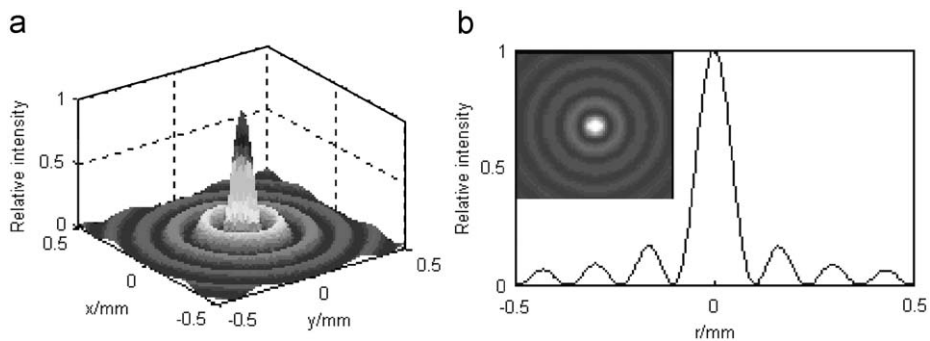


Fig. 3. Simulated intensity profiles of an ideal zero-order BBs: (a) three-dimensional distribution, and (b) beam pattern and radial distribution.

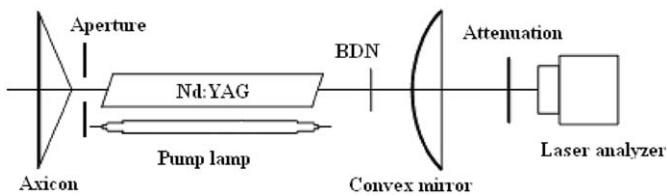


Fig. 4. Schematic illustration of the Nd:YAG laser cavity designed to generate zero-order BBs.

reflective mirror in the resonator. A bis-(4-dimethylaminodithiobenzol-nickel) (BDN) dye film is used as a Q-switched element for the generation of nanosecond pulse, and the output coupler is a convex spherical mirror which spherical surface is coated with reflectivity 50% in the laser wavelength. In order to prove the validity of theory, experimental elements keep same parameters with that of theory. Such a construction allowed us to reduce the number of reflecting surfaces inside the resonator.

To observe intensity distribution, we use a laser parameter analyzer (Taper Cam D-UCM-20-15) to record a digital patterns formation. Between the convex mirror and the laser analyzer, attenuation is placed to prevent the damage of the laser analyzer by a high power pulse.

#### 4. Experimental results

With the arrangement of Fig. 4, we obtained laser operation at one BBs pulse per second. The threshold voltage of BBs mode for this cavity is about 0.4 kV.

Theoretically speaking, the beam stays Bessel-like with an intense central spot in  $L$  and according to equation (6)  $L=25$  cm

was chosen in our experiment. The transverse intensity distributions at different propagation distances were captured by laser analyzer. Three typical patterns at  $z=8, 12, 22$  cm were shown. The three-dimensional intensity distribution and beam pattern and radial profile of the generated pulse at  $z=8$  cm ( $z=0$  at the out coupler) were shown in Figs. 5(a) and (b), beam pattern and radial profile of the generated pulse at  $z=12$  and  $22$  cm were shown in Figs. 5(c) and (d), respectively. Compared with simulated results shown in Fig. 2, the experimental results match well with the simulated results. We also placed the analyzer at  $z=2, 4, 6, 10, 25$  cm from the out coupler, the similar patterns were captured except the pattern is poor at  $z=25$  cm. So we conclude that the beam generated from the resonator is indeed approximate pulsed zero-order BBs.

The temporal profile of the output pulse was observed and captured with a fast photodiode and an HP54502A 400 MHz digitizing oscilloscope; Fig. 6 shows the oscilloscope trace of a single Q-switched pulse with a pulse width (FWHM) of 4.592 ns.

Finally, the energy of single BBs pulse was measured. In Fig. 7, the output energy of this experiment systems mentioned above is plotted, and the minimum energy of the pulse generated from this optic resonator is 0.8 mJ. So we can obtain the lowest power of one BBs pulse  $P=170$  kW. From Fig. 5(b), we measure the central spot size of the pulsed zero order BBs, which is approximately  $210 \mu\text{m}$  in the diameter. From the theory, the central spot diameter given in terms of the first irradiance zero may be written as

$$d_0 = 2 \frac{2.405}{k \sin \theta} = \frac{0.7659 \lambda}{(n-1)\alpha} \tag{11}$$

The value of  $d_0$  for  $\lambda=1.064 \mu\text{m}$ ,  $n=1.458$ , and  $\alpha=0.5^\circ$  is  $204 \mu\text{m}$ , which is  $6 \mu\text{m}$  smaller than the experimental result.

The stability of the laser was affected by the symmetry and length of the cavity and the axicon manufacture precision. Once

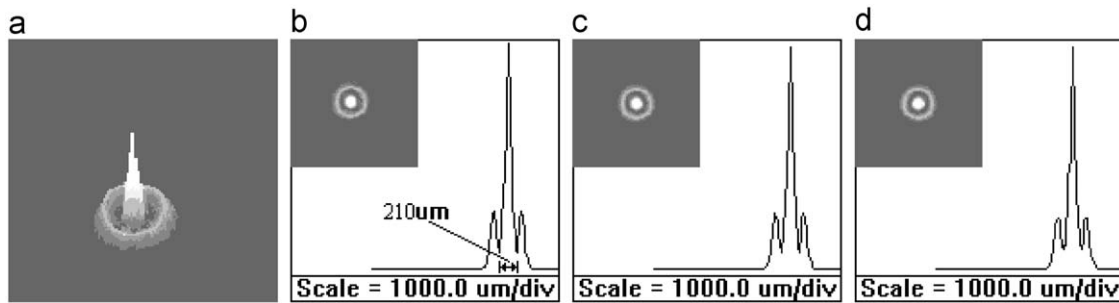


Fig. 5. Output intensity distribution captured by analyzer: (a) three-dimensional distribution at  $z=8\text{cm}$ , (b–d) beam pattern and radial distribution at  $z=8, 12,$  and  $22\text{ cm}$ .

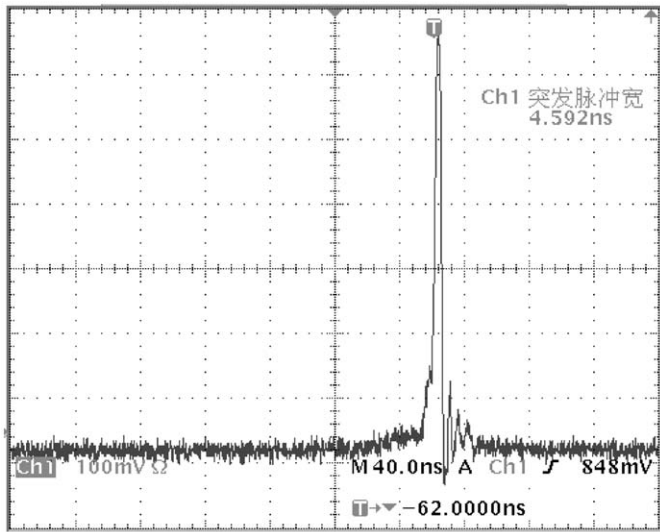


Fig. 6. Oscilloscope photograph of a single Q-switched zero-order BBs pulse (scale=40 ns/division).

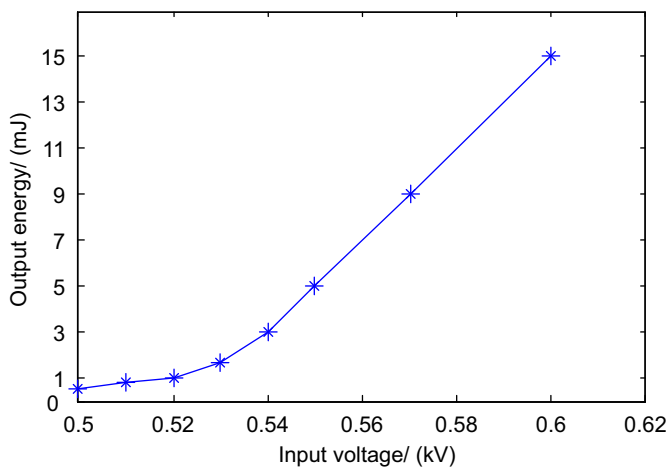


Fig. 7. BBs pulse energy and input voltage.

the symmetry of the resonator is broken by tilting axicon or manufactured error with non-circular symmetry or output coupler with a small angle, the higher-order cavity modes would be generated and the central spot and rings would be splitting [22]. The cavity length is depended on the beams radius  $a$  limited by the aperture and index  $n$  and base angle  $\alpha$  of axicon from Eq. (6). The change in cavity length will cause the change in cavity mode, laser threshold and propagation distance.

## 5. Conclusion

We have demonstrated a linear, and compact unstable resonator based on axicon to directly produce nanosecond zero-order BBs pulse. The experiment results were compared with the numerical simulation by the Fox–Li iterative method. This resonator is consisted of a conventional Nd:YAG laser unstable resonator with a flat mirror replaced by a glass axicon which possessing energy conversion efficiency almost 100% [23] and higher optical damage threshold. High power BBs pulses generated in our experiment are nearly non-diffraction and will find wide applications in the field of optical manufacturing and optical inspection. The resonator here we discussed is easy to realize, but its stability and the quality of the output beam should be further improved.

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