Market-Based Control of Shear Structures Utilizing Magnetorheological Dampers

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***Abstract*—The use of magnetorheological (MR) dampers for semi-active control of structures subject to seismic, wind, and/or other excitations has been an extensive field of study for over a decade. Many of the proposed feedback control laws have been based on modern linear systems control theory, e.g. linear quadratic Gaussian (LQG), H_2, or H_∞ control. Alternatively, this paper presents a nonlinear market-based controller (MBC) that explicitly handles the dynamic force saturation limits of MR dampers, a feature not available in the design of linear controllers. The MBC solution builds on an agent-based control (ABC) architecture with a diverse population of buying and selling agents capable of sensing and control respectively. These agents participate in a competitive market place trading control energy in a way that leads to a Pareto optimal allocation of control force during each control time step. The ABC architecture allows for easy implementation with inexpensive partially-decentralized large-scale wireless sensing and control networks. This novel controller is validated using a numerical simulation of a seismically excited six story shear structure with MR dampers at the base of V-braces installed on each story. The MBC is compared against a benchmark LQG controller in a variety of test cases including best case scenarios, robustness to agent failure, variations in ground excitation, variation in peak ground acceleration, and controller delays.**

I. Introduction

Control of the dynamic response of civil structures (e.g. bridges, buildings, and towers) has been extensively studied since the 1970's, yet a limited number of building owners are choosing structural control over more traditional passive design methodologies [1]. The high installation costs of the feedback control system components (i.e. sensors, actuators, centralized controller, and associated wiring) required to execute the control law are partially responsible. These costs can be reduced by implementing partially decentralized control architectures that utilize wireless controllers. Fortunately, the advances in low-cost microcontroller design and fabrication have recently led to the development of inexpensive wireless controllers with collocated sensing, actuation, communication, and computation abilities [2]. Lynch, Swartz, Yang, *et al*. have demonstrated in simulation and experimentation the ability of low cost wireless controllers, with varying degrees of decentralization, to effectively control civil structures using MR dampers [2-10].

Feedback structural control systems can be broadly grouped into two categories, active and semi-active. Active control systems (e.g. active-mass dampers) make use of actuators that directly apply forces on the structural system but utilize a substantial amount of power. Alternatively, semi-active systems indirectly apply forces on the structural system by changing member properties such as member stiffness or damping. Examples of semi-active devices include semi-active hydraulic dampers (SHD), variable-stiffness devices (VSD), variable-friction dampers, semi-active tuned mass dampers (STMD), electrorheological (ER) dampers, and magnetorheological (MR) dampers [1]. MR dampers are particularly well suited for use as semi-active devices in civil structures due to their low power requirements, small size, high dynamic range, and relatively quick dynamics [11].

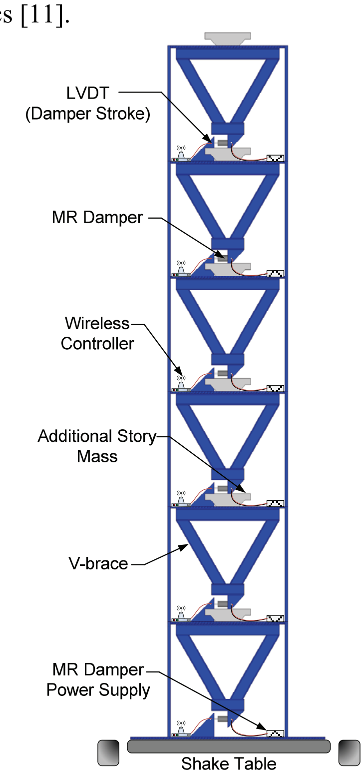

Fig. 1. 6-story partial-scale single-bay steel structure used for validation of MR damper control laws on the NCREE shake-table.

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The design of structures utilizing V-braces as a lateral force resisting system can be slightly modified to allow for the installation of MR dampers as shown in Fig. 1. When used in this manner, the number of MR dampers in a large structure may reach into the 100s. Optimally controlling such a system with a centralized controller would require extensive lengths of cables running to a central computer. If the central computer were to fail, the entire system would enter a passive state leading to a significant drop in system performance. Alternatively, the control architecture could be completely decentralized, without the need for expensive data cabling or the linchpin of a centralized computer. However decentralized controllers cannot offer the same level of system performance when compared to centralized controllers due to each decentralized controller's sparse information about the state of the whole system [12].

Partially decentralized control architectures can achieve a balance between centralization and decentralization. In wireless partially decentralized control, a separate controller is collocated with each actuator and utilizes its embedded sensing and computation along with a wireless transceiver to share information with other local controllers. In small structures with a reliable wireless communication environment, 'local' can mean all controllers in the network. However, in a large structure or for structures with an unfavorable wireless communication environment, 'local' can refer to only a few controllers within a reliable communication range. In order to achieve the best possible performance, the control architecture should be able to adapt to the time-varying definition of 'local'. This paper will investigate one such control architecture that has been inspired by the control of free market economies, referred to herein as market-based control (MBC).

In this study an MBC law is formulated to control the non-linear response of a six story single bay shear structure outfitted with MR dampers subject to seismic disturbances (see Fig. 1). The control law, abbreviated MR-MBC, is specifically formulated to account for the non-linear nature of the MR dampers as described by a Bouc-Wen hysteresis model. The paper concludes with the results of a numerical simulation comparing the MR-MBC law with both passive control and a benchmark LQG controller.

II. A Formulation Of Market Based Control For Control Of MR Dampers

The trade of goods and services has existed between humans for thousands of years. For most of that time there has been no central entity controlling the price of goods and the quantity that must be produced. Instead, each individual strives to maximize their utility with the limited amount of knowledge available to them as to how much of a good they should trade at the market price. A price is optimal to all agents in the market when there is equilibrium such that the price that buyers are willing to purchase goods at is equal to the price that suppliers are willing to manufacture enough goods to meet demand [13]. Since people have been trading for much longer then designing control systems, the theories of microeconomics are better established to explain the processes behind the distributed resource allocations. The analogy between the distributed resource allocation found in economies and that found in distributed control of dynamic systems is so strong that the well established microeconomic theories are the foundation for the new field of market-based control of dynamic physical [14] and computer [15] systems.

A. Control systems as a distributed allocation problem

Controllable dynamic systems can often be described by the discrete-time state space equation presented in (1), where the system has N dynamic degrees of freedom, M inputs $u(k)$, and time-steps k of length Δt. The state of the system can be fully described by the $Nx1$ state vector $x(k)$ at each time step. The evolution of the state of the system is a function of the current state, $x(k)$, current input, $u(k)$, and the initial state of the system x_0.

$$x(k+1) = f\big(x(k), u(k)\big), \qquad x(0) = x_0 \tag{1}$$

It is assumed that each controllable input is represented by an agent j that has a non-negative cost, K_j, associated with the production of each unit of production $c_j(k)$. Similarly, the n metrics to be controlled are each represented by an agent i that receives a non-negative utility, Φ_i, associated with each unit of a good $u_i(k)$ received by the agent.

$$K_j\left(c_j(k)\right) \geq 0 \;\forall\, j = 1, \dots, M \tag{2}$$

$$\Phi_i\big(u_i(k)\big) \geq 0 \;\forall\, i = 1, \dots, n \tag{3}$$

Therefore, the goal of the centralized resource allocation problem is the solution to the optimization problem

$$\begin{aligned} &\max J(u(k), c(k), x(k), k) \\ \text{s.t. } &\textstyle\sum_{i=1}^{n} u_i(k) = \sum_{j=1}^{M} c_j(k), \;\; c_j(k) \geq 0 \;\forall j. \end{aligned} \tag{4}$$

J is an objective function describing the efficiency of the consumption and production of a resource subject to the constraint that the sum of the resources consumed by consumers $i=1\dots n$, $u_i(k)$, is equal to the sum of the resources produced by producers $j=1\dots M$, $c_j(k)$. However due to the complex and often non-linear nature of large-scale control problems, the solution to (4) is often very difficult to find. Alternatively, Voos and Litz [14] proposed to individually maximize each agents objective function separately (5), a task that can be completed in real-time by agents using their onboard computing, wireless transceivers, and a set of market rules.

$$\begin{aligned} &\max J_1(c_1(k), k) \\ &\vdots \\ &\max J_M(c_M(k), k) \\ &\max J_1(u_1(k), x_1(k), k) \\ &\vdots \\ &\max J_n(u_n(k), x_n(k), k) \\ \text{s.t. } &\textstyle\sum_{i=1}^{n} u_i(k) = \sum_{j=1}^{M} c_j(k), \;\; c_j(k) \geq 0 \;\forall j \end{aligned} \tag{5}$$

Equation (5), while formulated as an optimal control problem, is also in the form of a special distributed resource allocation problem that has a set of Pareto optimal resource allocations at each step in time k. Pareto optimal solutions are those solutions to resource allocation problems that occur when all consumers and producers consume $(u^*_1(k), ..., u^*_n(k))$ and produce $(c^*_1(k), ..., c^*_M(k))$ respectively and there is no other allocation that would increase any agents objective function without simultaneously decreasing another agent's objective function [13].

The optimal solution to (5) can only be computed by the agents when they have been given a set of rules dictating how to trade in the market and how to maximize their objective function. First, the objective function of each agent must be formulated to map the current state of an agent to its desire to purchase or supply a quantity of a good at a particular price. Second, a set of rules must be given to each agent to optimize its objective function by trading with other agents. One set of rules could require each agent to send out its entire objective function to all other agents. Afterwards each agent solves the centralized resource distribution problem. If information transfer is limited, the rules could instead require each agent to send only a limited amount of information only to certain agents. In the case of limited information transfer, the convergence to the Pareto optimal allocation may require an iteration of negotiations between agents.

B. Single degree of freedom (SDOF) formulation

The goal of this paper is to present an MR-MBC law that can be implemented on a network of wireless control units utilizing MR dampers. This implementation environment, with limited communication and computational capacity, restricts the agents' ability to be omniscient and to solve the centralized control problem individually. Additionally, the communication rate of wirelessly networked controllers may not be rapid enough to allow for the market to iterate negotiations to quickly converge to a solution. One possible solution to these problems, which serves as the basis of this paper, is to allow the agents to use simple heuristics that permit each agent to approximately increase their objective function by estimating the environment in which they are acting. The formulation of the heuristics, rules, and objective functions will be presented for a single SDOF system with a single buyer and seller. The concepts will then be extended to more complex MDOF systems.

1) The Supplier's Utility

The goal of the supply agent, representing the MR damper, should be to minimize the amount of actuation supplied, and thus minimize the power consumed. The cost, in amperes, should increase as the specified amount of force increases up to some saturation limit. Unfortunately the force saturation limit of MR dampers is a dynamic property that changes with respect to the damper's velocity and hysteresis. For a supply agent to compute the current saturation limit it must employ a model of the nonlinear dynamics of the MR damper. While many models of MR dampers have been proposed, the supply agent will use a Bouc-Wen hysteretic model [16]. The Bouc-Wen model was first applied to model MR dampers by Spencer *et al.* [17] then adapted for use in real time systems by Lin [18]. The nonlinear force velocity relationship of the MR damper is defined as

$$F(V(t),t) = C\big(V(t)\big)\,\dot{y}(t) + z(t), \tag{6}$$

where $C(V)$ is the viscous damping coefficient as a function of command voltage, V, $y(t)$ is the differential displacement between the body of the MR damper and its shaft, F is the force in the damper, and $z(t)$ is the hysteresis force of order n as defined by (7).

$$\begin{aligned}\dot{z}(t) = {} & A\big(V(t)\big)\,\dot{y}(t) \\ & + \beta\big(V(t)\big)\,|\dot{y}(t)|\,|z(t)|^{n-1}\,z(t) \\ & + \gamma\big(V(t)\big)\,\dot{y}(t)\,|z(t)|^{n}\end{aligned} \tag{7}$$

The Euler method of integration is applied to (7) to compute a discrete time equation of the 2[rd] order hysteretic force, (8), that the supply agents can exploit at each time step.

$$\begin{aligned}z(k+1) = {} & z(k) + \Delta t\, A\big(V(k)\big)\,\dot{y}(k) \\ & + \Delta t\,\beta\big(V(k)\big)\,|\dot{y}(k)|\;|z(k)|\;z(k) \\ & + \Delta t\,\gamma\big(V(k)\big)\,\dot{y}(k)\,|z(k)|^2\end{aligned} \tag{8}$$

Before the model could be codified, the voltage dependent parameters C, A, β, and γ were identified for 10 different voltage level using the methods in [18] and then stored in a lookup table for use by the agents.

The cost of control force is based on the heuristic that an increase in voltage will increase the magnitude of the command force, up to the force produced by the maximum voltage $F(V_{max}, k)$ (abbreviated as F_{max}). From microeconomic theory the constraints in (9) are placed on the twice differentiable supplier's cost function $K_j(F_j(k+1))$. The supplier's cost function when defined by (10) quantifies the supplier's heuristic while also abiding by the constraints in (9) which helps to guarantee at least one Pareto optimal solution exists.

$$\begin{aligned}& \frac{dK_j\big(F_j(k+1)\big)}{dF_j(k+1)} > 0 \quad \text{and} \quad \left.\frac{dK_j}{dF_j}\right|_{F_j=0} = 0 \\ & \frac{d^2K_j\big(F_j(k+1)\big)}{dF_j^2(k+1)} \ge 0 \qquad \forall\, F_j(k+1) > 0\end{aligned} \tag{9}$$

$$\begin{aligned}\mathrm{K}_j\big(F_j(V(k+1),k+1\big) & \\ = {} & -\mu \ln\left(1 - \tfrac{F_j(V(k+1),k+1)}{F_j(V_{max},k+1)}\right) \\ & + \tfrac{\mu\, F_j(V(k+1),k+1)}{F_j(V_{max},k_1)}\end{aligned} \tag{10}$$

The cost function utilizes a one-step ahead prediction of the maximum possible control force magnitude to find the asymptote of the negative logarithmic relationship between a specified control force $F_j(V(k+1), k+1)$ and the cost K_j. The tuning variable μ is a non-negative real number that the designer chooses to adjust the rate of convergence to the asymptote. The goal of each supplier j is to maximize their profit by producing F_j units of control force at the market price p^* that solves

$$\max_{F_j \le F_{j,max}} p^*\, F_j - K_j\big(F_j\big). \tag{11}$$

2) The Buyer's Utility

The goal of the buyer in MR-MBC is to minimize the response of the structure by purchasing control force. In this study, the buyers are the wireless controllers measuring the response of the structure. The heuristic the agent uses to formulate the utility function says that an increase in the magnitude of control force, $F(k)$, should decrease the inter-story drift and velocity. The utility function, Φ, of the buyer as defined by (13) follows microeconomic theory in that it is a twice differentiable function bounded by the constraints in (12).

$$\frac{d\Phi\big(F(k)\big)}{dF(k)} > 0 \quad and\ \Phi(0) = 0$$
$$\frac{d^2\Phi\big(F(k)\big)}{dF(k)^2} < 0 \quad \forall\, F(k) > 0 \tag{12}$$

$$\Phi\big(F(k)\big) = -\frac{0.5\,F^2(k)}{T\,|y(k)| + Q\,|\dot{y}(k)|} + F(k)\,\tau\,w(k) \tag{13}$$

The amount of utility an agent receives from each unit of control force, F, increases as the inter-story drift or velocity increases. An increase in the buyer's wealth, $w(k)$, increases the utility received from each unit of control purchased. T, Q, and τ are non-negative constants that the control engineer can choose to adjust how much $y(k)$, $\dot{y}(k)$, and $w(k)$ affect the agents utility.

The buyer's utility function is designed to reduce the risk of structural damage (the inter-story drift term) and increase energy dissipation (the inter-story velocity term). It should be noted that the utility function is only based on a heuristic that an increase in control force magnitude will decrease these metrics; clearly, this may not hold for all possible system states. This inaccuracy has the advantages that the utility function is not based on a model (with possible modeling error) and is simple enough that agents can easily compute their own utility. Additionally, the simplicity of the utility function allows for it to be described at any point in time with only two values, $(T\,|y(k)| + Q\,|\dot{y}(k)|)$ and $(\tau\, w(k))$. This will aid the system in finding the market equilibrium.

3) Equilibrium

With the rules for the agents to formulate their utility functions set, the rules for the agents to communicate and agree on an equilibrium price must be specified. The market's Pareto optimal equilibrium point occurs when each agent cannot increase their own utility without also decreasing the utility of some other agent. The market achieves this equilibrium by establishing an equilibrium market price that is the only price that force can be traded. Just as in physics, equilibrium is the state of a system where opposing forces are balanced. In the case of markets, the forces are the buyer's push to make the prices lower, to increase their utility, and the supplier's push to raise the market price to increase revenue. Due to the physical constraints on the system all of the control force produced must be equal to the control force consumed (*e.g.* it is impossible for a damper to deliver a force without some part of the structure receiving that force). With these two constraints ($p_{supply} = p_{demand}$ and $F_{supply} = F_{demand}$) the equilibrium must occur where the supply and demand curves intersect. The supply and demand curves intersect at the solution of the optimization problem presented in (14).

$$\max_{F_{demand}\geq 0} \Phi(F_{demand}) - p^*\,F_{demand} + w$$
$$\max_{F_{supply}\geq 0} p^*\,F_{supply} - K\big(F_{supply}\big) \tag{14}$$
$$s.t. \quad F_{demand} = F_{supply} = F$$

The utility function of the buying agents was conveniently formulated such that the first maximization in (14) is the maximization of a 2nd-order polynomial that is strictly concave on the interval $0 \leq F < \infty$. Therefore the maximum is located where the derivative of the function being maximized is zero. Similarly, the second maximization in (14) is also strictly concave on the interval and the maximization can be found where the second derivative is zero. These two observations lead to the following simplification in finding F_{eq} and p_{eq}^*.

$$\frac{-F}{T\,|y| + Q\,|\dot{y}|} + \tau\,w \quad = p_{eq}^*$$
$$\frac{-\mu}{F_{eq} - F_{max}} + \frac{\mu}{F_{max}} \quad = p_{eq}^* \tag{15}$$

A solution must exist to the two simultaneous equations in (15) for $F \geq 0$ due to constraints (9) and (12). Therefore the Pareto optimal solution for the SDOF case of the MR-MBC is guaranteed and occurs when an equilibrium quantity (16) is traded at a price determined by (15) with $F = F_{eq}$. Fig. 2 shows that this equilibrium exists at the intersection of the supply and demand curves for some arbitrary state.

$$F_{eq} = \frac{a_1\,a_2\,F_{max} - \mu\,a_2 + F_{max}^2 + \sqrt{a_3}}{2\,F_{max}}$$
where:
$$a_1 = \tau\,w \quad a_2 = T\,|y| + Q\,|\dot{y}| \tag{16}$$
$$a_3 = a_1^2\,a_2^2\,F_{max}^2 - 2\,a_1\,a_2^2\,F_{max}\,\mu - 2\,a_1\,a_2\,F_{max}^3 + \mu^2 a_2^2 + 6\,\mu\,a_2\,F_{max}^2 + F_{max}^4$$

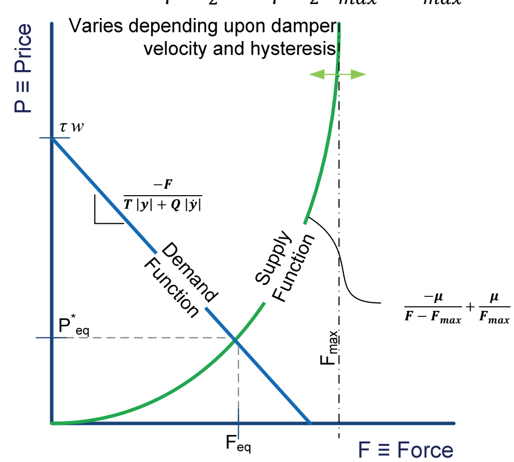

Fig. 2. Graphical depiction of the supply and demand curves for the SDOF MBC formulated above.

An individual agent is not able to locate the equilibrium price or control force since their local information is limited. To acquire the necessary information, the agents must trade information in messages with their neighbors in a negotiation process. The agents must communicate the messages over a wireless network; therefore the message size should be small, and more importantly the number of messages required to converge to equilibrium should be minimized.

The existence of an explicit function for (16) makes it possible for an agent to negotiate with messages, $M \in \{[\{S,D\}\ \mathbb{R}^+\ \mathbb{R}^+]\}$, only if every agent knows that every other agents' utility function is in the form of (10) or (13). For the supply agent, the message should be equal to (17), while the demand agent should send (18).

$$M_{supply} = [S \quad \mu \quad F_{max}] \tag{17}$$

$$M_{demand} = [D \quad (T\,|y| + Q\,|\dot{y}|) \quad \tau\, w] \tag{18}$$

Using (16) to compute equilibrium by sending messages (17) and (18) requires that the agents are cooperative in that they do not lie to other agents about their utility. The rules for each agent to execute within each control-step are as follows:

- R1. Every agent shall formulate their utility functions in the form of (10) or (13).
- R2. Every agent shall transmit a message M to every other agent in the form of (17) or (18).
- R3. Every agent shall compute the equilibrium market quantity by (16) and price by (15).

Since every agent is required to follow rules R1-R3, each agent will choose a Pareto optimal action by simply optimizing their own utility. As the control steps occur, the buying agent takes part of their initial endowment of wealth $w(0)=w_0$ and gives it to the supplier. Eventually the buying agents will run out of wealth if there is not some method for redistributing the wealth from the suppliers back to the buyers. To alleviate this problem for SDOF MR-MBC the buyer receives a re-endowment at the beginning of each step equal to the amount spent during the previous step.

C. Extension to multiple degrees of freedom (MDOF)

The advantage of MR-MBC over some other control laws is the ability of MBC to control a system with a large number of actuators. For this reason the SDOF formulation presented in the previous subsection should be extended to a MDOF case. The agents in the MDOF MR-MBC utilize utility functions based on the same heuristics used by agents in SDOF MR-MBC. A shear structure with N stories and an MR damper installed at the base of a V-brace on every floor as depicted in Fig. 1 is one example to be used to study MR-MBC. The MR damper of floor j is selling in the market the control force $F_j(k)$ at each control step k. Similarly each floor has an agent i measuring inter-story drift $y_i(k)$ and velocity $\dot{y}_i(k)$ that will try to maximize its utility $\Phi_i(k)$ by purchasing control force from its supplier.

For simplicity sake, an explicit formulation between the minimization of $y_i(k)$ and $\dot{y}_i(k)$ w.r.t. the control force at any arbitrary floor j was not developed for use in the agents demand function. Instead, the buyers use a heuristic that says only the force from supplier j with $j = i$ is capable of effecting $y_i(k)$ or $\dot{y}_i(k)$. The buying agent also assumes that the action made by any other supplier has no effect on $y_i(k)$ or $\dot{y}_i(k)$. Similar to the heuristic simplification used in the SDOF formulation, this heuristic minimizes the amount of computation required by the agents. The heuristic is quantified by (19) as the buyers utility function.

$$\Phi_i\left(F_j(k)\right) = \frac{-0.5\, F_j^2(k)}{T|y_i(k)| + Q|\dot{y}_i(k)|} + F_j(k)\tau\, w_i(k) \qquad \forall\, i \le n, where\ j = i \tag{19}$$

The utility function for the sellers in a MDOF MR-MBC is equal to that of the suppliers in the SDOF implementation of MR-MBC presented in (10). The MDOF and SDOF implementations are similar in that each market has only a single buyer and a single seller, except that there are *6* of these markets occurring simultaneously and independently. The biggest change from SDOF MR-MBC is the wealth redistributed after each control step. When the buyers are initially deployed, they are all given an initial wealth w_0. However, if some agents' purchases are better at controlling the overall response of the structure, then it would make sense to distribute more wealth to those agents such that in the next step, the agents have more wealth to better control the response of the whole structure. The wealth redistribution rule takes the total amount of wealth transferred in all markets Γ in the previous step and distributes $s\cdot\Gamma$ directly back to the agents that spent it. The remaining is sent to each agent i as a payment P_i governed by (20) representing how much agent i helped to improve the total state of the structure.

$$P_i = (1-s)\,\Gamma\,\left(L\,\frac{\bar{Y}_i + \left|\min_{n\le N}\bar{Y}_n\right|}{1+N\left|\min_{n\le N}\bar{Y}_n\right|} + M\,\frac{\dot{\bar{Y}}_i + \left|\min_{n\le N}\dot{\bar{Y}}_n\right|}{1+N\left|\min_{n\le N}\dot{\bar{Y}}_n\right|}\right)$$

where:

$$0 \le s \le 1$$
$$\bar{Y}_i = \frac{y_i^2(k) - y_i^2(k-1)}{\sum_{n=1}^{N}\left(y_n^2(k) - y_n^2(k-1)\right)} \tag{20}$$
$$\dot{\bar{Y}}_i = \frac{\dot{y}_i^2(k) - \dot{y}_i^2(k-1)}{\sum_{n=1}^{N}\left(\dot{y}_n^2(k) - \dot{y}_n^2(k-1)\right)}$$

The execution of each control step starts with the agents formulating their utility function either in the form of (10) or (19). Buying agents transmit a message in the form of (18) to their collocated supplier, while the suppliers send their message in the form of (17). The equilibrium point in each market as computed by (16) determines the amount of control force the supplier should generate, and the amount of wealth the buyer should pay. At the conclusion of each step, every supplier should inform the wealth distribution agent of the amount of wealth received such that it can be redistributed according to (20). When all the agents follow the rules described in this formulation of MR-MBC, a locally Pareto optimal control force is generated at each MR damper. Also, wealth is transferred between markets such that more efficient markets may obtain more wealth to purchase more control force over the time trajectory of the system.

III. NUMERICAL SIMULATION AND VALIDATION

The National Center for Research on Earthquake Engineering (NCREE) in Taipei, Taiwan has graciously provided the authors with a model of the six-story steel structure as seen in Fig. 1 that was installed on their shaking table in March of 2010. The partial scale, single-bay structure has a floor-to-floor height of 1.0 m, bays 1.0 m square, and 15 cm x 2.5 cm steel columns oriented in their weak flexural direction. Lord Corporation RD-1005-3 MR dampers are installed on each story at the base of H100x100x6x8 steel V-braces. A 0-0.8 V signal amplified by a 24 V, 2 A power amplifier is used for semi-active control of the six MR dampers. The MR dampers have a maximum force capacity of ±2.0 kN and a maximum stroke of 20 mm. NCREE has also supplied the authors with coefficients of a discrete time Bouc-Wen model used to simulate the response of the MR dampers.

A. Benchmark Controllers

Due to the difficultly of mathematically proving bounds for non-linear market-based control laws, the effectiveness of MR-MBC was determined by a parametric study of the simulated controlled response of the NCREE structure subjected to single direction ground motions. The MR-MBC controlled response was compared against the response of the structure under identical ground motions and network environments when controlled by a clipped-optimal linear quadratic Gaussian (LQG) controller and maximum (i.e. passive on) and zero (i.e. passive off) damper voltages.

Clipped-optimal LQG was chosen as the benchmark control law due to its previously successful use as a control law for the wireless control of structures similar to the one used in this study [7, 19-20].

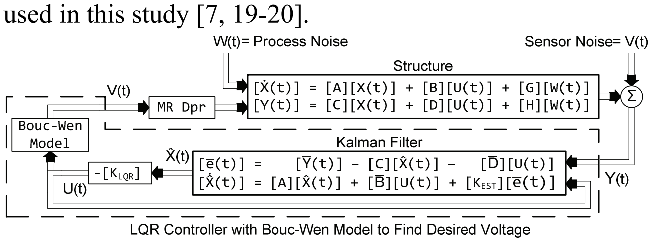

Fig. 3. Block diagram of LQG control of MR dampers.

The LQG control law, schematically shown in Fig. 3, generates an optimal desired control force *U(t)* for the structure represented by a state variable model (SVM) with story position and velocity relative to ground as states *X(t)* and absolute story acceleration as the output *Y(t)*. In most physical environments the system states are immeasurable and must be estimated by a Kalman filter with noisy absolute acceleration input returning an estimated state $\hat{X}(t)$. The LQR was designed to minimize inter-story drift, $\bar{Y}(t)$, instead of the more typical *X(t)* such that the benchmark controller and the MR-MBC were minimizing the same metrics. The weighting variables *Q* and *R* were specified according to the method proposed by Bryson and Ho [21]. The proposed method specifies *Q* and *R* as diagonal matrices of the percent change from the maximum expected values of $\bar{Y}(t)$ and *U(t)* respectively. A non-negative constant *ρ* can be multiplied with *Q* to allow for a greater or less weighting of the response versus control force.

$$J(U) = \int_0^\infty (\bar{Y}^T Q\, \bar{Y} + U^T R\, U) dt \qquad (21)$$

$$U(t) = -K_{LQR}\, \hat{X}(t) \qquad (22)$$

The cost function is minimized by the control law (22) where K_{LQR} is related to the solution S of the algebraic Riccati equation (23) by $K_{LQR} = \hat{R}^{-1} (\bar{B}^T S)$. The MATLAB® Control System Toolbox [22] was utilized to compute the solutions to the Riccati equations to solve for the LQR and estimator gains.

$$A^T S + SA - (S\,\bar{B} + N)\,\hat{R}^{-1}\left(B^T S + \hat{N}^T\right) + \hat{Q} = 0$$
$$\text{Where:} \quad \begin{bmatrix} \hat{Q} & \hat{N} \\ \hat{N}^T & \hat{R} \end{bmatrix} = \begin{bmatrix} C^T & 0 \\ \bar{D}^T & I \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} C & \bar{D} \\ 0 & I \end{bmatrix} \qquad (23)$$

The output of the control law *U(t)* must be compared against the estimated damper force as computed by the Bouc-Wen model for 10 discrete damper voltages. The voltage that results in an estimate closest to the desired control force is applied to the damper. The control law becomes suboptimal when the desired control force *U(t)* varies in sign or magnitude from the actual damper force.

B. Metrics

To judge the performance of the MR-MBC against the three benchmark controllers, eight cost functions *J1-J8* are proposed similar to those developed in [23]. The first three measure the root-mean-squared (rms) response of the structure, while the 4th through 7th measure the peak response. The 8th cost function is a measure of the mean electrical power consumed by the MR damper averaged over the simulation length. Formulas for the cost functions are presented in Table I.

TABLE I
COST FUNCTIONS TO COMPARE CONTROLLER RESPONSES

$J1^{a,b} = \dfrac{y_{rms:Controlled}}{y_{rms:Uncontrolled}}$	$J5^{c} = \dfrac{\max_{Floor,t} \lvert \ddot{x}_{controlled} \rvert}{\max_{Floor,t} \lvert \ddot{x}_{uncontrolled} \rvert}$
$J2^{a,c} = \dfrac{\ddot{x}_{rms:Controlled}}{\ddot{x}_{rms:Uncontrolled}}$	$J6^{d} = \max_{Floor,t} \lvert F \rvert$
$J3^{a,d} = F_{rms}$	$J7^{c,e} = \max_{t} \dfrac{\lvert \ddot{x}_{controlled} \boldsymbol{W} \rvert}{\lvert \ddot{x}_{uncontrolled} \boldsymbol{W} \rvert}$
$J4^{b} = \dfrac{\max_{Floor,t} \lvert y_{controlled} \rvert}{\max_{Floor,t} \lvert y_{uncontrolled} \rvert}$	$J8^{f} = \dfrac{1}{6\, t_{end}\, V_{max}} \sum_{n=1}^{6\ Floors} \left(\int_0^{t_{end}} (V_n(t)) dt \right)$

The comparison between controlled and uncontrolled cases occur while keeping the ground motion magnitude and type the same across simulations.

[a] $x_{rms} = \left(\frac{1}{(\#\ Floors)(\#\ simulation\ steps)} \right) \sqrt{\sum_{n=1}^{\#\ Floors} \sum_{k=1}^{\#\ sim\ steps} \left(x_n(k) \right)^2}$
where $x_n(k)$ is some metric measured at floor *n* at step *k*
[b] y ≡ Inter-story drift
[c] $\ddot{x}$ ≡ Absolute story acceleration
[d] F ≡ Measured force on the MR damper
[e] **W** ≡ Seismic mass vector. $\ddot{x}$ **W** ≡ Base shear
[f] V ≡ Voltage supplied to MR damper

C. Simulation Cases and Results

The simulations were run with MATLAB® in a simulation environment developed by the authors. The simulator keeps separate the parameters governing the simulation (e.g. structure properties, sensor noise, and non-deterministic controller delay) and the parameters used by the controllers. This separation makes certain that agents cannot receive information about other agents unless the rules described above are followed to do so. The separation also allows for the simulator to utilize a set of 'real' system properties, while the controllers used estimated properties. In this way one can easily study the effect of model uncertainty on the controller. The parametric study to determine the effectiveness of MR-MBC considered the effect of six different environmental parameters; controller type, failure of sensors & actuators, variation in ground motion magnitude and type, controller delay, and model uncertainty.

The first study compares the response of the structure under the best-case scenario of controller environment. In this case the controllers are given the same model parameters that are used in the simulation. The controllers are allowed to update their output voltage at 100 Hz. The structure is excited by a unidirectional 100 Gal peak-ground-acceleration (p.g.a.) record of the 1940 El Centro (Imperial Valley Irrigation District Station) earthquake.

Study 2 evaluates the response of the controllers to a power failure on the 3rd floor resulting in zero damper voltage and no available 3rd accelerometer reading. Since the MR-MBC is agent based, the remaining agents can quickly realize that agent 3 has stopped responding and refrain from including the lost agent in their calculations. However due to the limited resources available to wireless LQG agents, they would be unable to recomputed the solution to the Riccati equation to find the optimal control force under failure. Instead, the LQG controller must rely on the Kalman filter that assumes a measurement of zero and tries to estimate the current state with the zero measurement.

The third study simulates the structure excited by the ground motions other than El Cento, the ground motion for which the MR-MBC and LQG controllers were tuned. The 1999 Chichi, Taiwan station TCU076 and white noise acceleration were chosen as the alternative ground motions. Both records were scaled to a p.g.a. of 100 Gal.

Since both the control law and actuators in MR-MBC are non-linear, it is of interest to study the response of the controlled structure excited by motions of different magnitude. Study 4 examines the response of the structure to a test similar to study one with p.g.a. of 50 Gal and 200 Gal respectively.

Wireless control networks will inherently have a delay larger than their wired counterparts. To study the effect of the cost saving adoption of wireless controllers, the fifth set of simulations increases the controller delays to 50 Hz and then to 20 Hz. Previous work has shown that wireless control networks of this size can currently communicate at rates up to 50 Hz [7].

The final series of simulations studies the effect of model uncertainty on the response of the different control laws. In this study an extra 100 kg is added to each story in the simulated structure, however the controllers are unaware of the added mass. This amounts to approximately a 15% modeling error in the mass of the structure. The structure is excited by the same 100 Gal motion as study one.

A total of twenty-two simulations were conducted as part of the parametric study. The three plots in Fig. 4 represent a portion of the results that capture well the performance of the four different control laws in a variety of tests. Due to the non-linear characteristics of both the MR dampers and the MR-MBC a quantitative analysis of the results is difficult and not required. Instead, a qualitative analysis is presented in the conclusion describing the efficacy of MR-MBC.

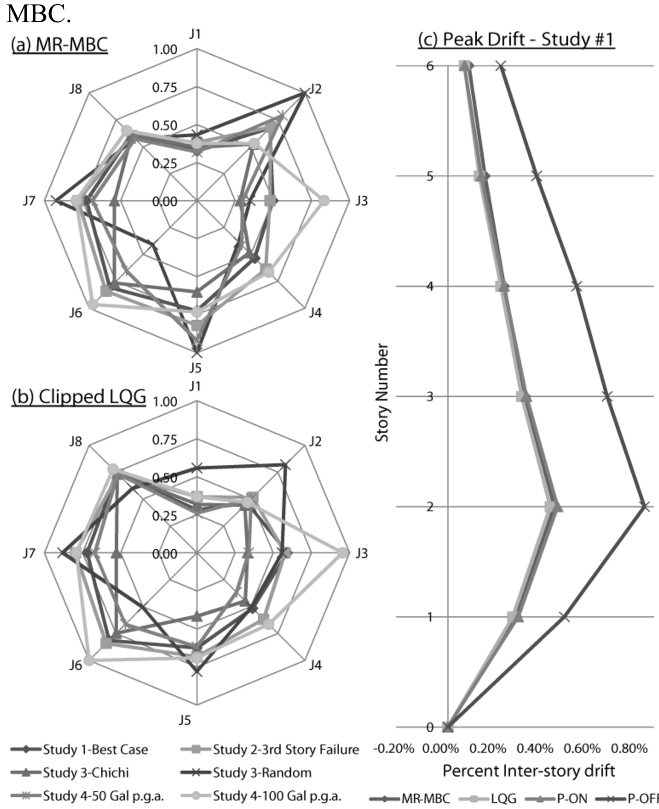

Fig. 4. The star plots (a) and (b) depict partial results of the parametric study for the MR-MBC and LQG controller respectively. The cost functions *J1-J8*, defined in Table I, were scaled such that the largest value of each cost function across all tests has a value equal to one. The qualitative performance of a controller is loosely related to the area enclosed by the octagon whose corners are positioned on the eight cost function axes. (c) shows that the MR-MBC, LQG, and passive on control laws all perform similarly in controlling inter-story drift.

IV. Conclusion

This paper presented a non-linear closed-loop MR-MBC controller for the semi-active control of shear structures using MR dampers. A parametric study was undertaken to compare the effectiveness of the MR-MBC against a LQG controller and two passive open-loop controllers. The results of the parametric study, qualitatively presented in Fig. 4, show that MR-MBC can successfully limit the response of structures during seismic events of different types and magnitudes. In the metrics that both the MR-MBC and LQG controllers were designed to control, *J1, J4, J8*, the MR-MBC was shown to be just as effective and in some cases marginally better than LQG control.

Not apparent in the results presented was the problems associated with designing a linear LQG controller for control of a highly nonlinear system. During study 1, the LQG controller frequently desired over 400% of the possible control force and desired more control force than was achievable over 60% of the time. The clipping of the LQG controller led to suboptimal performance. However, the MR-MBC always knew the force capacity of each MR damper, due to the Bouc-Wen model employed by the suppliers, resulting in power savings over the LQG during most tests.

In conclusion, a decentralized architecture for semi-active control of civil structures has been proposed. While the heuristics utilized by the agents sacrifice some accuracy for computational efficiency, the resulting decentralized controllers perform on par with centralized LQG solutions. Future work may include the development of physics based heuristics that account for effect of a single damper on every story of the structure. This would require the solutions to the decentralized resource allocation problem referred to by economists as the allocation of public goods. The development of these stronger heuristics along with the experimental validation in a realistic packet-loosing wireless network may show the true efficacy of MR-MBC.

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