Chapter 1
The Foundation: Logic and Proof

1.1 Propositional Logic

1. Introduction

- Logic is the basis for mathematical reasoning.
  - Application:
    1. verification the correctness of programs;
    2. ……………………

1. Introduction

- Example 1
  - Algebraic Laws of Programming Languages

Language:

Skip

x:=e

$P \triangleleft b \triangleright Q$

while b do P

Semantics using Logic Formulae

Laws:

(1) x:=e;x:=f(x) = x:=f(x)

(2) (P;Q):R = P;(Q;R)

(3) $P \triangleleft b \triangleright P = P$

(4) $(P \triangleleft b \triangleright Q) \triangleleft c \triangleright R$

$= P \triangleleft b/\backslash c \triangleright (Q \triangleleft c \triangleright R)$

(5) while b do P = if b then (P ; while b do P) else Skip

2.Propositions (命题)

- Definition of proposition
  - A proposition is a declarative statement that is true or false, but not both. (即:表示判断的语句称为命题)
- Example 1 (see page 2)
  - All the following declarative sentences are propositions:
    1. Washington, D.C. is the capital of the United States of America.
    2. Toronto is the capital of Canada.
    3. 1+1=2.
    4. 2+2=3.
    - Propositions 1 and 3 are true, whereas 2 and 4 are false

2.Propositions (命题)

- Example 2 (not propositions)
  - Consider the following sentences.
    1. What time is it now?
    2. Read it carefully?
    3. $x+1 = 2$
    4. $x+y = z$
    - Sentences 1 and 2 are not declarative sentences.
    - Sentences 3 and 4 are neither true or false.
- The truth value of proposition is true, denoted by T.
- The false value of proposition is false, denoted by F.

2.Propositions (命题)

- Negation of a Proposition (“非”)
  - Definition 1
    - Let p be a proposition. The statement “It is not the case that p” is another proposition, called the negation of p.

1. The negation of p is denoted by $\neg p$.
2. The proposition “$\neg p$” is read “not p”.

- Truth Table

p	$\neg p$
T	F
F	T

2.Propositions (命题)

- Negation of a Proposition (“非”)
  - Example 3
    - Find the negation of “Today is Friday.”
    - Solution:
      1. “It is not the case that today is Friday,”
      2. or “Today is not Friday,”
      3. or “It is not Friday today.”

2.Propositions (命题)

- Conjunction (“并且”又称”合取”)
  - Definition
    - Let p and q be propositions. The proposition “p and q”, denoted as “p ∧ q” is the proposition that is true when both of them are true and is false otherwise. The proposition “p ∧ q” is called the conjunction of p and q.
    - Truth Table

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

2.Propositions (命题)

- Conjunction
  - Example 5 (page 4)
    - Find the conjunction of the propositions p and q where p is the proposition “Today is Friday” and q is the proposition “It is raining today.”
    - Solution:

      1. $p \wedge q$ is the proposition “Today is Friday and it is raining today.”
      2. When is $p \wedge q$ true?

2.Propositions (命题)

- Disjunction (“或者”又称”析取”)
  - Definition
    - Let p and q be propositions. The proposition “p or q”, denoted as $p \vee q$, is the proposition that is false when p and q are both false and true otherwise. The proposition $p \vee q$ is called the disjunction of p and q.
    - Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

2.Propositions (命题)

- Example 6
  - Find the disjunction of the propositions p or q where p is the proposition “Today is Friday” and q is the proposition “It is raining today.”
  - Solution:
    - $p \vee q$ is the proposition “Today is Friday or it is raining today.”
    - When is $p \vee q$ true?

2.Propositions (命题)

- Exclusive (“异或”)
  - Definition
    - Let p and q be propositions. The exclusive or of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise
    - Truth Table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

2.Propositions (命题)

- Implication (“蕴含”)/Conditional Statement
  - Definition
    - Let p and q be propositions. The implication $p \rightarrow q$ is the proposition that is false when p is true and q is false, and is true otherwise.
    - In this implication p is called the hypothesis (or antecedent or premise) and q is called the conclusion (or consequence)
    - Truth Table

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

2.Propositions (命题)

- Remark
  - a variety of terminology to express $p \to q$ (page 6).
  - When is $p \to q$ false?
    - How about the case that p is false?
  - $q \to p$ is called the converse of $p \to q$ (逆命题).
  - $\neg q \to \neg p$ is called contrapositive of $p \to q$ (逆否命题).
  - $\neg p \to \neg q$ is called inverse of $p \to q$ (否命题).
- Example 9 (see page 8)
  - What are the converse, contrapositive, inverse of the implication “ The home team wins whenever it is raining.”?
  - Solution:
    - The implication can be rewritten as: “If it is raining, then the home team wins” Then …………………

2.Propositions (命题)

- Biconditional (“当且仅当”又称”等价”)
  - Definition
    - Let p and q be propositions. The biconditional $p \leftrightarrow q$ is the proposition that is true when p and q have the same truth values, and is false otherwise.
    - Truth Table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

2.Propositions (命题)

- $p \leftrightarrow q$ -------------------”p if and only if q”
- $p \leftrightarrow q$ has the same truth table of $(p \to q) \wedge (q \to p)$
- Example 10 (see page 9)
  - Let p be the statement “You can take the flight” and let q be the statement “You buy a ticket.”
  - Then $p \leftrightarrow q$ is the statement “You can take the flight if and only if you buy a ticket.”

2.Propositions (命题)

- Truth Tables of compound Propositions
  - Example 11
    - Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$
    - Solution: The Truth table is:

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

3.Precedence of Logical Operations

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

- Example:
  1. $p \wedge q \vee r$ means $(p \wedge q) \vee r$ rather than $p \wedge (q \vee r)$
  2. $p \vee q \rightarrow r$ is the same as $(p \vee q) \rightarrow r$.
  - Remark: We will use parentheses when the order of the conditional operator and biconditional operator is at issue.

4. Translating English Sentences

- Example 12
  - How can this English sentence be translated into a logical expression.
    - “You can access the internet from the campus only if (page 6) you are a computer science major or you are not a freshman (新生).”
  - Solution:
    - a------”You can access the internet from the campus.”
    - b------”You are a computer science major.”
    - c------”You are a freshman.”
    - Then this sentence can be expressed as $a \rightarrow (c \vee \neg b)$

4. Translating English Sentences

- Example 13
  - How can this English sentence be translated into logical expression?
    - “You cannot ride the roller coaster (过山车) if you are under 4 feet tall unless you are older than 16 years older.”
  - Solution:
    - q--------”You can ride the roller coaster.”
    - r---------”You are under 4 feet tall.”
    - s--------”You are older than 16 years older.”
    - Then the sentence can be translated to

$$(r \land \neg s) \rightarrow \neg q.$$

Homework

- Page 16～19
  - 2, 4, 6, 8, 12, 26, 28, 30, 32,