Chapter 1
The Foundation: Logic and Proof

1.5 Rules of Inference

1. Introduction

- Please read the book (page 63)
  - 补充如下中文定义:
    - 设H1,H2,...,Hn,C是一组逻辑公式，当且仅当H1 H2 ... Hn→C是永真公式称C是一组前提H1,H2,...,Hn的有效结论。
    - 判断有效结论的过程就是论证过程。
    - 使用的方法：真值表法、直接证法、间接证法。

2.Valid Arguments in Propositional Logic

- Modus Ponens (假言推理)
  - Hypotheses (前提)------- p， $p \to q$
  - Conclusion （结论)------- q
  - also written as: p

$$
\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}
$$

- The Correctness of Modus Ponens (假言推理的正确性)
  - $(p \land (p \to q)) \to q$ is a tautology(永真公式)

2.Valid Arguments in Propositional Logic

- Valid Arguments (有效推理)
  - Definition (page 64)
    - An argument form is called valid if whenever all the hypotheses (或称premise, 前提) are true, the conclusion (结论) is also true.
    - Consequently, showing that q logically follows from the hypotheses p1, p2, ..., pn is the same as showing that the implication $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is tautology (永真公式)

3. Rules of Inferences in Propositional Logic

- Table 1 (page 66)
  - Modus ponens (假言推理)

    p

    $p \rightarrow q$

    $\therefore q$

  - Modus tollens (拒取式)

    $\neg q$

    $p \rightarrow q$

    $\therefore \neg p$

3. Rules of Inferences in Propositional Logic

- Table 1 (page 66)
  - Hypothetical syllogism (假言三段论)

    $p \to q$

    $q \to r$

    $\therefore\ p \to r$

  - Disjunctive syllogism (析取三段论)

    $p \vee q$

    $\neg p$

    $\therefore\ q$

3. Rules of Inferences in Propositional Logic

- Table 1 (page 66)
  - Addition (附加)

    p

    $\therefore p \vee q$

  - Simplification (化简)

    $p \wedge q$

    $\therefore p$

3. Rules of Inferences in Propositional Logic

- Table 1 (page 66)
  - Conjunction (合取)

$$
\begin{array}{l}
p \\
q \\
\hline
\therefore p \land q
\end{array}
$$

  - Resolution

$$
\begin{array}{l}
p \lor q \\
\neg p \lor r \\
\hline
\therefore q \lor r
\end{array}
$$

3. Rules of Inferences in Propositional Logic

- Example 5 (page 67)
  - State which rule of inference in the argument:
    - “If it rains today, then we will not have a barbecue (烧烤) today. If we do not have a barbecue today, then we will have a barbecue tomorrow. Therefore, if it rains today, we will have a barbecue tomorrow.”

3. Rules of Inferences in Propositional Logic

- Example 5(cont.)
  - Solution:
    - p------It is raining today.
    - q------We will not have a barbecue today.
    - r------We will have a barbecue tomorrow.
  - Then this argument is of the form:

$$p \rightarrow q$$

$$q \rightarrow r$$

$$p \rightarrow r$$

4. Using Rules of Inference to Build Argument

- Example 6 (page 67)
  - Show that the hypotheses
    1. “It is not sunny this afternoon and it is colder than yesterday.”
    2. “We will go swimming only if it is sunny.”
    3. “If we do not go swimming, then we will take a canoe (独木舟) trip.”
    4. “If we take a canoe trip, then we will be home by sunset.”
  - lead to the conclusion
    1. “We will be home by sunset.”

4. Using Rules of Inference to Build Argument

- Example 6
  - Solution:
    - p------It is sunny this afternoon.
    - q------It is colder than yesterday.
    - r------We will go swimming.
    - s------We will take a canoe trip.
    - t-------We will be home by sunset.
    - The hypotheses are: $\neg p \land q$, $r \rightarrow p$, $\neg r \rightarrow s$, $s \rightarrow t$
    - The conclusion is: t

4. Using Rules of Inference to Build Argument

- Example 6
  - Solution:

Step	Reason
1. $\neg p \land q$	Hypothesis
2. $\neg p$	Simplification using (1)
3. $r \to p$	Hypothesis
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \to s$	Hypothesis
6. s	Modus tollens using (4) and (5)
7. $s \to t$	Hypothesis
8. t	Modus tollens using (6) and (7)

4. Using Rules of Inference to Build Argument

- Example 6
  - Solution(用中文):

步骤	理由
1. ¬p ∧ q	前提引入
2. ¬p	(1)化简
3. r→p	前提引入
4. ¬r	(2)(3)拒取式
5. ¬r→s	前提引入
6. s	(4)(5)假言推理
7. s→t	前提引入
8. t	(6)(7)假言推理

4. Using Rules of Inference to Build Argument

- Example 7 (page 67)
  - Show that the hypotheses
    1. "If you send me an e-mail message, then I will finish writing the program."
    2. "If you do not send me an e-mail message, then I will go to sleep early."
    3. "If I go to sleep early, then I will wake up feeling refreshed."
  - lead to the conclusion
    1. "If I do not finish writing the program, then I will wake up feeling refreshed."

4. Using Rules of Inference to Build Argument

- Example 7 (page 67)
  - Solution:
  1. p-----You send me an e-mail message.
  2. q-----I will finish writing the program.
  3. r------I will go to sleep early.
  4. s-----I will wake up feeling refreshed.
  - The hypotheses are: $p\to q$, $\neg p\to r$, $r\to s$
  - The conclusion is: $\neg q\to s$

4. Using Rules of Inference to Build Argument

- Example 7 (page 67)
  - Solution:

Step		Reason
1.	$p \to q$	Hypothesis
2.	$\neg q \to \neg q$	Contrapositive of (1)
3.	$\neg p \to r$	Hypothesis
4.	$\neg q \to r$	Hypothetical Syllogism using (2) and (3)
5.	$r \to s$	Hypothesis
6.	$\neg q \to \neg s$	Hypothetical Syllogism using (4) and (5)

4. Using Rules of Inference to Build Argument

- Example 7 (page 67)
  - Solution（用中文）:

步骤	理由
1. p→q	前提引入
2. ¬q→¬q	(1) 置换 (或写为: (1)逆否命题)
3. ¬p→r	前提引入
4. ¬q→r	(2)(3)假言三段论
5. r→s	前提引入
6. ¬q→¬s	(4)(5)假言三段论

4.命题逻辑推理理论的其他方法(补充)

- 附加前提法
  - 分析：

$$(A1 \land A2 \land \ldots \land Ak) \rightarrow (A \rightarrow B)$$

$$\equiv \neg (A1 \land A2 \land \ldots \land Ak) \lor (\neg A \lor B)$$

$$\equiv (\neg (A1 \land A2 \land \ldots \land Ak) \lor \neg A) \lor B$$

$$\equiv \neg (A1 \land A2 \land \ldots \land Ak \land A) \lor B$$

$$\equiv (A1 \land A2 \land \ldots \land Ak \land A) \rightarrow B$$

所以我们可以把A作为附加前提

4.命题逻辑推理理论的其他方法(补充)

- 例1：证明下面推理为有效推理：
  - 前提： $p \to q$， $(r \vee \neg s) \to \neg q$
  - 结论： $p \to (\neg r \wedge s)$
  - 证明：
    - 方法1：附加前提法

步骤		理由
1.	p	附加前提引入
2.	$p \to q$	前提引入
3.	q	(1)(2)假言推理
4.	$(r \vee \neg s) \to \neg q$	前提引入
5.	$q \to \neg(r \vee \neg s)$	(4)置换
6.	$\neg(r \vee \neg s)$	(3)(5)假言推理
7.	$\neg r \wedge s$	(6)置换

4.命题逻辑推理理论的其他方法(补充)

- 例1：证明下面推理为有效推理：
  - 前提： $p \to q$， $(r \lor \neg s) \to \neg q$
  - 结论： $p \to (\neg r \land s)$
  - 证明：
    - 方法2：本题也可以用直接法

步骤		理由
1.	$(r \lor \neg s) \to \neg q$	前提引入
2.	$q \to \neg(r \lor \neg s)$	(1)置换
3.	$q \to (\neg r \land s)$	(2)置换
4.	$p \to q$	前提引入
5.	$p \to (\neg r \land s)$	(3)(4)假言三段论

4.命题逻辑推理理论的其他方法(补充)

- 归谬法
  - 分析:

    $(A1 \wedge A2 \wedge ... \wedge Ak) \rightarrow B$ (若为永真公式)

    $\equiv \neg (A1 \wedge A2 \wedge ... \wedge Ak) \vee B$

    $\equiv \neg (A1 \wedge A2 \wedge ... \wedge Ak \wedge \neg B)$

  - 括号里面的公式(红色)为永假公式, 其中:¬B称为结论否定引入

4.命题逻辑推理理论的其他方法(补充)

- 例2：证明下面推理为有效推理：
  - 前提： p→¬q， r→q，r
  - 结论： ¬p
  - 证明
    - 方法1：用归谬法

步骤	理由
1． p	结论否定引入
2． p→¬q	前提引入
3． ¬q	(1)(2)假言推理
4． r→q	前提引入
5． ¬r	(3)(4)拒取式
6． r	前提引入
7． ¬r ∧ r	(5)(6)合取

本题也可以用直接证明法证明

5. Rules of Inference for Quantified Statements

- Universal instantiation (全称量词消去规则, 即:UI规则)

  $\forall x\, P(x)$

  $\therefore P(c)$

  - Here, c is a particular member of the domain
  - Example:
    - We can conclude from the statement "All women are wise" that "Lisa is wise" where, Lisa is female.

5. Rules of Inference for Quantified Statements

- Universal generalization (全称量词引入规则, UG规则)

  P(c) for an arbitrary c

  $\therefore \forall x\ P(x)$

- Existential instantiation (存在量词消去规则, EI规则)

  $\exists x\ P(x)$

  $\therefore$P(c) for some element c

- Existential generalization (存在量词引入规则, EG规则)

  P(c) for some element c

  $\exists x\ P(x)$

5. Rules of Inference for Quantified Statements

- Example 12 (page 70)
  - Show that the premises
    1. “Everyone in the discrete mathematics class has taken a course in computer science”
    2. “Marla is a student in this class”
  - Imply
    1. “Marla has taken a course in computer science”

5. Rules of Inference for Quantified Statements

- Example 12
  - Solution:
    - D(x)-----”x is in this discrete mathematics class”
    - C(x)-----”x has taken a course in computer science”
    - Premises: $\forall x\,(D(x)\rightarrow C(x))$, D(Marla)
    - Conclusion: C(Marla)

5. Rules of Inference for Quantified Statements

- Example 12

Step	Reason
1. $\forall x\ (D(x) \rightarrow C(x))$	Premise
2. $D(Marla) \rightarrow C(Marla)$	Universal instantiation from (1)
3. D(Marla)	Premise
4. C(Marla)	Modus tollens using (2) and (3)

  - 或用中文表示如下：

步骤	理由
1. $\forall x\ (D(x) \rightarrow C(x))$	前提引入
2. $D(Marla) \rightarrow C(Marla)$	(1)UI规则
3. D(Marla)	前提引入
4. C(Marla)	(2)(3)假言推理

5. Rules of Inference for Quantified Statements

- Example 13 (page 71)
  - Show that the premises
    1. "A student in this class has not read the book" and
    2. "Everyone in this class passed the first exam"
  - imply the conclusion
    1. "Someone who passed the first exam has not read the book."

5. Rules of Inference for Quantified Statements

- Example 13
  - Solution:
    1. C(x)------”x is in this class”
    2. B(x)------”x has read the book”
    3. P(x)------”x passed the first exam”
  - The premises:
    1. $\exists x\ (C(x) \land \neg B(x))$
    2. $\forall x\ (C(x) \rightarrow P(x))$
  - The conclusion: $\exists x\ (P(x) \land \neg B(x))$

5. Rules of Inference for Quantified Statements

- Example 13
  - Solution

Step	Reason
1. $\exists x\,(C(x) \land \neg B(x))$	Premise
2. $C(a) \land \neg B(a)$	Existential instantiation from (1)
3. $C(a)$	Simplification from (2)
4. $\forall x\,(C(x) \rightarrow P(x))$	Premise
5. $C(a) \rightarrow P(a)$	Universal instantiation from (4)
6. $P(a)$	Modus ponens from (3) and (5)
7. $\neg B(a)$	Simplification from (2)
8. $P(a) \land \neg B(a)$	Conjunction from (6) and (7)
9. $\exists x\,(P(x) \land \neg B(x))$	Existential generation from (8)

5. Rules of Inference for Quantified Statements

- Example 13
  - Solution(用中文)

步骤	理由
1. $\exists x\ (C(x) \land \neg B(x))$	前提引入
2. $C(a) \land \neg B(a)$	(1)UI规则
3. $C(a)$	(2)化简
4. $\forall x\ (C(x) \rightarrow P(x))$	前提引入
5. $C(a) \rightarrow P(a)$	(4) UI规则
6. $P(a)$	(3)(5)假言推理
7. $\neg B(a)$	(2)化简
8. $P(a) \land \neg B(a)$	(6)(7)合取
9. $\exists x\ (P(x) \land \neg B(x))$	(8) EG规则

Homework

- page
  - 12，24，26，28，30
  - 补充题目