Chapter 2
Basic Structures: Sets, Functions, Sequences and Sum

2.1 Sets

1. Introduction

- Definition 1
  - A set is an unordered collection of objects.
- Definition 2
  - The objects in a set are also called the elements or members, of the set. A set is said to contain its elements

1. Introduction

- How to describe a set?
  - The first way: listing all the members of a set
  - Examples (page 112)
    - The set V of all vowels (元音字母) in the English alphabet: V={a, e, i, o, u}
    - The set of odd positive integers less than 10: O={1, 3, 5, 7, 9}
    - A set can contain some unrelated elements: {a, 2, Fred, New Jersey}
    - The set of positive integers less than 100:{1, 2, 3, ..., 99}
    - Some important sets (page 112~113): N, Z, Z+, Q, R

1. Introduction

- The equation of two sets
  - Definition 3 (page 79)
    - Two sets are equal if and only if they have the same elements.
    - Example 6 (page 113)
      - $\{1, 3, 5\} = \{3, 5, 1\} = \{1, 3, 3, 3, 5, 5, 5, 5\}$

1. Introduction

- How to describe a set?
  - The second way: Using set builder notation
  - Example
    - the set of all odd positive integers less than 10

    O={x|x is an odd positive integer less than 10}
- Venn diagram (文氏图)
  - Universal set U (全集):containing all the objects under consideration
  - Example
    - Venn diagram for the set of vowels (page 114)
    - Solution: See blackboard.

1. Introduction

- Empty set (空集)
  - { } or $\varnothing$ How about { $\varnothing$ }?
- Subset (子集)
  - Definition 4 (page 114)
    - The set A is said to be a subset of B if and only if every element of A is a also an element of B. We use the notation $\subseteq$ to indicate that A is a subset of the set B.
      $A \subseteq B$ iff $\forall x\ (x \in A \rightarrow x \in B)$

1. Introduction

- Theorem 1 (page 115)
  - For any set S, (a) $\varnothing \subseteq S$ (b) $S \subseteq S$
- Proper subset (真子集)
  - A is a proper subset of B if and only if A is a subset of B but that $A \neq B$.
  - The notation: $A \subset B$
- One way to show that two sets are equal
  - $A=B$ iff $A \subseteq B$ and $B \subseteq A$

1. Introduction

- Cardinality (基数)
  - Definition 5 (page 116)
    - Let S be a set. If there are exactly n distinct elements in S where n is nonnegative integer, we say that S is a finite set and that n is the cardinality of S. The cardinality of S is denoted by |S|.
  - Example 9 (page 116)
    - Let A be the set of odd positive integers less than 10. Then |A|=5.

1. Introduction

- Definition 6 (page 116)
  - A set is said to be infinite if it is not finite.
- Example12 (page 116)
  - The set of positive integers is infinite.

2. The Power Set (幂集)

- Definition 7 (page 116)
  - Given a set S, the power set of S is the set of all subsets of the set S. The power set of S is denoted by P(S).
- Example 13
  - What is the power set of {0, 1, 2}?
    - Solution: See page 116.
  - What is power set of the empty set?
  - What is the power set of the set $\{\emptyset\}$?
    - Solution: See page 117.

3. Cartesian product (笛卡儿乘积)

- Ordered n-tuple (a1, a2, ..., an)
  - Definition 8 (page 117)
    - The ordered n-tuple (a_1, a_2, ..., a_n) (有序n元组) is the ordered collection that has a_1 as its first element, a_2 as its second element, ... , a_n as its nth element.
    - Equality of two ordered n-tuples

      (a_1, a_2, ..., a_n) = (b_1, b_2, ..., b_n) iff $a_i = b_i$ for i=1,2,......,n

3. Cartesian product (笛卡儿乘积)

- Cartesian product of two sets
  - $A \times B = \{ (a,b) \mid a \in A \land b \in B \}$
  - Example 16 (page 118)
    - A={1, 2} and B={a, b, c}
    - How about $A \times B$ and $B \times A$?
      - Solution: See page 118.

3. Cartesian product (笛卡儿乘积)

- Cartesian product of A_1, A_2, ..., A_n
  - $A_1 \times A_2 \times \ldots\ldots \times A_n$ = $\{(a_1, a_2, \ldots\ldots, a_n) \mid a_i \in A_i \text{ for } i=1, 2, \ldots, n\}$
- Example 18 (page 118)
  - A={0,1}
  - B={1,2}
  - C={0,1,2}
  - How about $A \times B \times C$?

4 Using set notation with quantifiers

- $\forall x \in S\ P(x)$ ------- $\forall x\ (x \in S \rightarrow P(x))$
- $\exists x \in S\ P(x)$ ------- $\exists x\ (x \in S \wedge P(x))$
- Example 19 (page 119)
  - What do the statements $\forall x \in R\ (x2 \geqslant 0)$ and $\exists x \in Z\ (x2 \geqslant 1)$ mean?
  - Solution: See book.

Homework

- page 119~121
  - 4, 8, 18, 20, 22, 28, 34