Chapter 5 Counting

5.1 The Basics of Counting

1. Introduction

- Example 0 (page 335)
  - A password on a computer system consists of six, seven, or eight characters. Each of these characters must be a digit or a letter of the alphabet. Each password must contain at least one digit. How many such passwords are there?

2. Basic Counting Principles

- The Product Rule (乘法法则)
  - Suppose that a procedure can be broken into a sequence of two tasks. If there are n1 ways to do the first task and n2 ways to do the second task after the first task has been done, then there are n1n2 ways to do the procedure.
- Examples
  - Example 2 (page 336)
    - The chair of an auditorium (礼堂) are to be labeled with a letter and a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?
    - Solution: See blackboard or book.

2. Basic Counting Principles

- Examples
  - Example 3 (page 336)
    - There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?
    - Solution: See blackboard or book.

2. Basic Counting Principles

- Extended Version of the Product Rule
  - Suppose that a procedure is carried out by performing the tasks T_1, T_2, ..., T_m in sequence. If task T_i can be done in n_i ways after tasks T_1, T_2, ..., and T_{i-1} have been done, then there are $n_1 \times n_2 \times \ldots \times n_m$ ways to carry out the procedure.
- Examples
  - Example 4 (page 336)
    - How many different bit strings are there of length seven?
    - Solution: See blackboard or book

2. Basic Counting Principles

- Examples
  - Example 5 (page 336)
    - How many different license plate are available if each plate contains a sequence of three letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)
    - Solution: See blackboard or book.
  - Example 6 (page 336) Counting functions
    - How many functions are there from a set of m elements to one with m elements.
    - Solution: See blackboard or book.

2. Basic Counting Principles

- Examples
  - Counting one-to-one functions
    - How many one-to-one functions are there from a set with m elements to one with n elements?
    - Solution: See blackboard or book.
  - Example 7 and 8 (page 303, 304)
    - Please read them by yourself.
  - Example 10 (page 338)
  - Counting Subsets of a Finite Set
    - Use the product rule to show that the number of different subsets of a finite set S is $2^{|S|}$.
    - Solution: See blackboard or book.

2. Basic Counting Principles

- The Product Rule from Viewpoint of Cartesian Product

  $|A_1 \times A_2 \times ... A_m| = |A_1| \times |A_2| \times ... \times |A_m|.$

2. Basic Counting Principles

- The Sum Rule (求和法则)
  - If a first task can be done in n1 ways and a second task in n2 ways, and if these tasks cannot be done at the same time, then there are n1+n2 ways to do one of these tasks.
  - Example 11 (page 338)
    - Suppose that either a member of the mathematics faculty or a student who is a mathematics major is chosen as a representative to a university committee. How many different choices are there for this representative if there are 37 members of the mathematics faculty and 83 mathematics major.
    - Solution: 37+83

2. Basic Counting Principles

- Extended Version of the Sum Rule
  - Suppose that the task T_1, T_2, ..., Tm can be done in n_1, n_2, ..., n_m ways, respectively, and no two of these tasks can be done at the same time. Then the number of ways to do one of these tasks is $n_1+n_2+...+n_m$.
  - Examples
  - Example 12 (page 339)
    - A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. How many possible projects are there to choose from
    - Solution:23+15+19
  - Example 13 Please read it by yourself.

2. Basic Counting Principles

- The Sum Rule from the Viewpoint of Sets
  - If A_1, A_2, ..., A_m are disjoint sets, then the number of the elements in the union of these sets is the sum of the numbers of elements in them.

$$|A_1 \cup A_2 \cup \dots \cup A_n| = |A_1| + |A_2| + \dots + |A_m|$$

3. More Complex Counting Problems

- Remark (备注):
  - Many complicated counting problems can be solved using both of these rules.
- Example 14 (page 340)
  - In a version of the computer language BASIC, the name of a variable is a string of one or two alphanumeric (文字数字的) characters, where uppercase and lowercase letters are not distinguished. Moreover, a variable name must be begin with a letter and must be different from the five strings of two characters that are reserved for programming use.
  - How many different variable names are there in the version of BASIC?
  - Solution: See blackboard or book.

3. More Complex Counting Problems

- Example 15 (page 340)
  - Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many different passwords are there?
  - Solution: See blackboard or book

4. The Inclusion-Exclusion Principle

- Example 17 (page 342)
  - How many bit strings of length eight either start with a 1 bit or end with two bits 00?
  - Solution: See blackboard or book.

4. The Inclusion-Exclusion Principle

- Simple Inclusion-Exclusion Principle
  - A_1-----T_1 ways to select an element from A_1
  - A_2-----T_2 ways to select an element from A_2??
  - How many ways to select an element from $A_1 \cup A_2$?
  - $| A_1 \cup A_2 | = |A_1| + |A_2| - |A_1 \cap A_2|$

5. Tree Diagram

- Example 19
  - How many bit strings of lengths of four do not have two consecutive 1s?
  - Solution: See blackboard or book.
- Example 19 (page 343)
  - A playoff ((双方得分相同时的)最后决赛) between two teams consists of at most five games. The first team that wins three grams win the playoff. In how many different ways can the playoff occur?
  - Solution: See blackboard or book.

5. Tree Diagram

- Example 20 (page 344)
  - Suppose that “I Love New Jersey” tee shirts come in five different sizes: S, M, L, XL, and XXL. Further suppose that each size comes in four colors, white, red, green, and black, except for XL, which comes only in read, green, and black, and XXL, which comes only in green and black.
    - How many different shirts does a souvenir shop have to stock to have at least one of each available size and color of the tee shirt?
    - Solution: See blackboard or book.

Homework

- Page 344~347
  - 16, 22(a)(b)(c), 24, 28, 34, 36, 38, 40