Chapter 5 Counting

5.3 Permutations and Combinations

1. Introduction

- Example 1 (page 355)
  - Please read the book.
- Permutations (排列)
  - Definition (page 355)
    - A permutation of a set of distinct objects is an ordered arrangements of these objects.
    - An ordered arrangement of r elements of a set is called an r-permutation.
  - Example 2
    - Let S={1,2,3}. The arrangement 3,1,2 is a permutation of S. The arrangement 3,2 is a 2-permutation of S.

2. Permutations

- Theorem 1
  - The number of r-permutations of a set with n distinct elements is

    $P(n,r) = n(n-1)(n-2)\ldots(n-r+1)$
  - Proof
    - See blackboard or book.
  - We can also have:
    - $P(n,r) = n! / (n-r)!$

2. Permutations

- Example 4 and 5 (page 356)
  - Please read them by yourself.
- Example 6
  - Suppose that a saleswoman has to visit eight different cities. She must begin her trip in a specified city, but she can visit the other seven cities in any order she wishes. How many possible orders can the saleswoman use when visiting these cities?
  - Solution: 7!

2. Permutations

- Example 7 (page 357)
  - How many permutations of the letters ABCDEFGH contains the string ABC?
  - Solution:
    - 6!
  - Way:
    - six objects

      -------the block ABC and the individual D,E,F,G, and H

3. Combinations

- Definition (page 357)
  - An r-combination of elements of a set is an unordered selection of r elements from the set.
- Example 9 (page 357)
  - Let S be the set {1,2,3,4}. Then {1,3,4} is a 3-combination from S.
- Example 10
  - Please read it by yourself.

3. Combinations

- Theorem 2 (page 358)
  - The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \leqslant r \leqslant n$, equals

    $C(n,r) = n! / (r!(n-r)!)$
  - Proof:
    - Step1: First we can get

      $$P(n,r) = C(n,r) \times P(r,r)$$
    - Step 2: Then we can get our result.

3. Combinations

- Example 11 (page 258)
  - How many poker hands of five cards ca deck of 52 can be dealt from a standard deck of 52 cards? Also, how many ways there to select 47 cards from a standard deck of 52 cards?
  - Solution:

$$C(52,5) = 52!/(5!\ 47!)$$

$$= (52 \cdot 51 \cdot 50 \cdot 49)/(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$$

3. Combinations

- Corollary 2
  - Let n and r be nonnegative integers with r⩽n.

    Then C(n,r) = C(n,n-r)
- Example 12 and 13 (page 360)
  - Please read them by yourself.
- Example 14
  - How many bit strings of length n contain exactly r 1s?
  - Solution:
    - See blackboard or book.(注意直观理解)

3. Combinations

- Example 15 (page 360)
  - How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department, if there are nine faculty members of the mathematics department and 11 of the computer science department?
  - Solution:

    $C(9,3) \times C(11,4)$

Homework

- Page 360~362
  - 20, 22, 26