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Steady State Solution and Stability of an Age-Structured MSIQR Epidemic Model

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ABSTRACT

The importance of epidemiology in our life has stimulated researchers to extend the classic Susceptibles-Infectives-Removed (SIR) model to sophisticated models by including more factors in order to give detailed transmission dynamics of epidemic diseases. However, the integration of the quarantine policy and age-structure is less addressed. In this work we propose an age-structured MSIQR (temporarily immune-susceptibles-infectives-quarantined-removed) model to study the impact of quarantine policies on the spread of epidemic diseases. Specifically, we investigate the existence of steady state solutions and stability property of the proposed model. The derived explicit expression of the basic reproductive number shows that the disease-free equilibrium is globally asymptotically stable if, and that the unique endemic equilibrium exists if. In addition, the stability conditions of the endemic equilibrium are derived.

KEYWORDS

Epidemic Model, Quarantine, Age-Structure, Basic Reproductive Number, Endemic Equilibrium, Stability

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