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# THERMAL SCIENCE

## International Scientific Journal

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### THE HEAT-BALANCE INTEGRAL METHOD BY A PARABOLIC PROFILE WITH UNSPECIFIED EXPONENT: ANALYSIS AND BENCHMARK EXERCISES

#### ABSTRACT

The heat-balance integral method of Goodman has been thoroughly analyzed in the case of a parabolic profile with unspecified exponent depending on the boundary condition imposed. That the classical Goodman's boundary conditions defining the time-dependent coefficients of the prescribed temperature profile do not work efficiently at the front of the thermal layers if the specific parabolic profile at issue is employed. Additional constraints based on physical assumption enhance the heat- -balance integral method and form a robust algorithm defining the parabola exponent. The method has been compared by results provided by the Veinik's method that is by far different from the Goodman's idea but also assume formation of thermal layer penetrating the heat body. The method has been demonstrated through detailed solutions of 4 1-D heat-conduction problems in Cartesian co-ordinates including a spherical problem (through change of variables) and over-specified boundary condition at the face of the thermal layer.

#### KEYWORDS

[heat-balance integral method](#), [heat-conduction](#), [parabolic profile](#), [unspecified exponent](#), [additional constraints](#), [boundary conditions](#)

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