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Stochastic Modeling of the Equilibrium Speed-Density Relationship

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Abstract

Fundamental diagram, a graphical representation of the relation among traffic flow, speed, and density, has been the foundation of traffic flow theory and transportation engineering for many years. For example, the analysis of traffic dynamics relies on input from this fundamental diagram to find when and where congestion builds up and how it dissipates; traffic engineers use a fundamental diagram to determine how well a highway facility serves its users and how to plan for new facilities in case of capacity expansion. Underlying a fundamental diagram is the relation between traffic speed and density which roughly corresponds to drivers' speed choices under varying car-following distances. First rigorously documented by Greenshields some seventy-five years ago, such a relation has been explored in many follow-up studies, but these attempts are dominantly deterministic in nature, i.e. they model traffic speed as a function of traffic density. Though these functional speed-density models are able to coarsely explain how traffic slows down as more vehicles are

crowded on highways, empirical observations show a wide-scattering of traffic speeds around the values predicted by these models. In addition, functional speed-density models lead to deterministic prediction of traffic dynamics, which lack the power to address the uncertainty brought about by random factors in traffic flow. Therefore, it appears more appropriate to view the speed-density relation as a stochastic process, in which a certain density level gives rise not only to an average value of traffic speed but also to its variation because of the randomness of drivers' speed choices. The objective of this dissertation is to develop such a stochastic speed-density model to better represent empirical observations and provide a basis for a probabilistic prediction of traffic dynamics. It would be ideal if such a model is formulated with both mathematical elegance and empirical accuracy. The mathematical elegance of the model must include the features of: a single equation (single-regime) with physically meaningful parameters and must be easy to implement. The interpretation of empirical accuracy is twofold; on the one hand, the mean of the stochastic speeddensity model should match the average behavior of the empirical equilibrium speeddensity observations statistically. On the other hand, the magnitude of traffic speed variance is controlled by the variance function which is dependent on the response. Ultimately, it is expected that the stochastic speed-density model is able to reproduce the wide-scattering speed-density relation observed at a highway segment after being calibrated by a set of local parameters and, in return, the model can be used to perform probabilistic prediction of traffic dynamics at this location. The emphasis of this dissertation is on the former (i.e. the development, calibration, and validation of the stochastic speed-density model) with a few numerical applications of the model to demonstrate the latter (i.e. probabilistic prediction). Following the seminal Greenshields model, a great variety of deterministic speeddensity models have been proposed to mathematically represent the empirical speeddensity observations which underlie the fundamental diagram. Observed in the existing speed-density models was their deterministic nature striving to balance two competing goals: mathematical elegance and empirical accuracy. As the latest development of such a pursuit, we show that the stochastic speed-density model can be developed through discretizing a random traffic speed process using the Karhunen- Lo`eve expansion. The stochastic speed-density relationship model is largely motivated by the prevalent randomness exhibited in empirical observations that mainly comes from drivers, vehicles, roads, and environmental conditions. In a general setting, the proposed stochastic speed-density model has two components: deterministic and stochastic. For the deterministic component, we propose to use a family of logistic speed density models to track the average trend of empirical observations. In particular, the five-parameter logistic speed-density model arises as a natural candidate due to the following considerations: (1) The shape of the five-parameter logistic speed-density model can be adjusted by its physically meaningful parameters to match the average behavior of empirical observations. Statistically, the average behavior is modeled by the mean of empirical observations. (2) A three-parameter and four-parameter logistic speed-density model can be obtained by reducing the shape or scale parameter in the five-parameter model, but the counter-effect is the loss of empirical accuracy. (3) The five-parameter model yields the best accuracy compared to three-parameter and four-parameter model. The magnitude of the stochastic component is dominated by the variance of traffic speeds indexed by traffic density. The empirical traffic speed variance increases as density increases to around 25 - 30 veh/km, then starts decreasing as traffic density gets larger. It has been verified by empirical evidence that traffic speed variation shows a parabolic shape which makes the proposed variance function in a suitable formula to model its variation. The variance function is dependent on the logistic speed-density relationship with varying model parameters. A detailed analysis of empirical traffic speed variance can be found in Chapter 6. Modeling results show that by taking care of second-order statistics (i.e., variance and correlation) the proposed stochastic speed-density model is suitable for describing the observed phenomenon as well as for matching the empirical data. Following the results, a stochastic fundamental diagram of traffic flow can be established. On the application side, the stochastic speed-density relationship model can potentially be used for real-time on-line prediction and to explain phenomenons in a similar manner. This enables dynamic control and management systems to anticipate problems before they occur rather than simply reacting to existing conditions. Finally, we will summarize our findings and discuss our future research