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An inexpensive estimate of failure probability for high-dimensional systems with uncertainty ☆

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Abstract

The failure probability of a system at an uncertain state can be estimated within a precise confidence interval using the Monte-Carlo sampling technique. Using this approach, the number of system parameters may be arbitrarily large, and the system may be non-linear and subject to random noise. For a given confidence level and interval, the number of required simulations can be exactly computed using the Beta Distribution. When failure probabilities are on the order of 1 – 10%, this technique becomes very inexpensive. In particular, *100 simulations are always sufficient* for a failure estimate with a confidence interval of +/−10% at a 95% confidence level.

In an engineering development process, this estimate limits the number of trials required to assess the robustness or reliability of high-dimensional and non-linear systems. When simulations are expensive, for example in vehicle crash development, using such a rule to minimize the number of trials can greatly reduce the expense and time invested in development.

Highlights

► Failure estimates may be computed with very few sample points for any system. ► Bounds on failure rate are provided by Bayesian analysis of Monte Carlo sampling. ► Number of samples needed is not affected by high dimensionality or non-linearity. ► Only 100 sample points are needed for an estimate error of 10% with 95% confidence.

Keywords

Robustness; Reliability; High-dimensional systems; Non-linear systems; Crash analysis; Bayesian inference

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Figures and tables from this article:

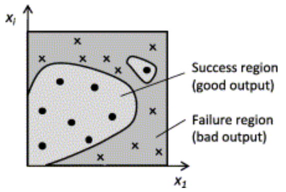

Fig. 1. The input space may be divided into a success and a failure region, where $z < z_c$ and $z > z_c$, respectively. Input points resulting in good outputs are shown as circles, and those resulting in bad outputs are shown as *x*’ s. For a uniform distribution of the input parameters, the probability of success will be equal to the good fraction of the input space volume.

Figure options

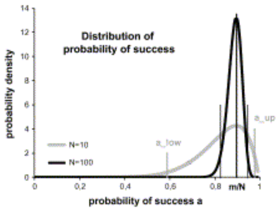

Fig. 2. Probability distributions of possible probabilities of success for an observed frequency of success *m*/*N* = 0.9, based on a set of 10 and a set of 100 samples. The confidence intervals are defined by a_{low} and a_{up}, shown as vertical lines.

Figure options

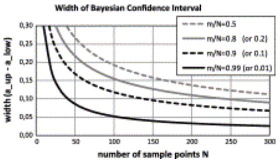

Fig. 3. Convergence of confidence interval with increasing sample size for a confidence level of 95%. The confidence interval converges most rapidly for observed success frequencies *m*/*N* close to 1 or 0.

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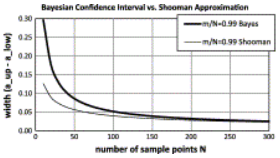

Fig. 4. Comparison of the exact confidence interval calculated using the Bayesian approach, versus the Shooman approximation. Shooman's estimate matches the exact (Bayesian) case well except where the underlying probability of success happens to be high and relatively few sample points have been taken.

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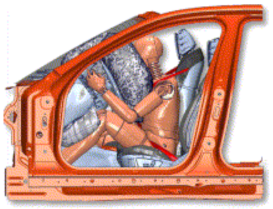

Fig. 5. Example of a crash analysis problem. In a front crash, the dummy is decelerated by the restraint systems such as airbags and seat belts. They are designed such that the loads on the dummy do not exceed critical threshold values. This may happen when the force exerted by the restraint system is too large, or as a result of contact with the interior parts of the vehicle such as the steering wheel.

Figure options

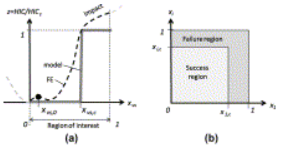

Fig. 6. (a) Relationship between HIC value *z* and vent size x_{vs}. The nominal design state is $x_{vs,0}$. The curves labeled *FE* and *model* represent the responses of a detailed Finite Element simulation and a binary-valued substitute model, respectively. For $x_{vs} > x_{vs,0}$ the airbag becomes too soft, and the passenger impacts the vehicle's interior. (b) Multi-dimensional representation of an input space. On the success region B_p, the system response is good, i.e. $z < z_c$.

Figure options

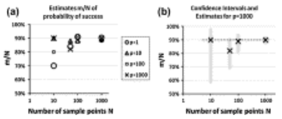

Fig. 7. (a) Estimates *m*/*N* of the probability of success. The underlying model has a probability of success of *a* = 0.9. (b) The

gray bars show the 95%-confidence intervals and the estimate of probability of success for samples with $p = 1000$ input parameters.

Figure options

☆ The ideas and opinions expressed in this paper are the authors' own, and do not in any way represent the views of GE Global Research, Germany, the General Electric Company or the BMW Group.

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