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## Local limit theorem for large deviations and statistical box-tests

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**Abstract:** Let  $n$  particles be independently allocated into  $N$  boxes, where the  $i$ -th box appears with the probability  $a_i$ . Let  $\mu_r$  be the number of boxes with exactly  $r$  particles and  $\mu = [\mu_{r_1}, \dots, \mu_{r_m}]$ . Asymptotical behavior of such random variables as  $N$  tends to infinity was studied by many authors. It was previously known that if  $a_i$  are all upper bounded and  $n/N$  is upper and lower bounded by positive constants, then  $\mu$  tends in distribution to a multivariate normal law. A stronger statement, namely a large deviation local limit theorem for  $\mu$  under the same condition, is here proved. Also all cumulants of  $\mu$  are proved to be  $O(N)$ .

Then we study the hypothesis testing that the box distribution is uniform, denoted  $h$ , with a recently introduced box-test. Its statistic is a quadratic form in variables  $\mu - \mathbf{E}\mu(h)$ . For a wide area of non-uniform  $a_i$ , an asymptotical relation for the power of the quadratic and linear box-tests, the statistics of the latter are linear functions of  $\mu$ , is proved. In particular, the quadratic test asymptotically is at least as powerful as any of the linear box-tests, including the well-known empty-box test if  $\mu_0$  is in  $\mu$ .

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