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## On the Amortized Complexity of Zero Knowledge Protocols for Multiplicative Relations

*Ronald Cramer and Ivan Damgard and Valerio Pastro*

**Abstract:** We present a protocol that allows to prove in zero-knowledge that committed values  $x_i, y_i, z_i$ ,  $i=1, \dots, l$  satisfy  $x_i y_i = z_i$ , where the values are taken from a finite field  $\mathbb{K}$ , or are integers. The amortized communication complexity per instance proven is  $O(\kappa + 1)$  for an error probability of  $2^{-1}$ , where  $\kappa$  is the size of a commitment. When the committed values are from a field of small constant size, this improves complexity of previous solutions by a factor of  $l$ . When the values are integers, we improve on security: whereas previous solutions with similar efficiency require the strong RSA assumption, we only need the assumption required by the commitment scheme itself, namely factoring. We generalize this to a protocol that verifies  $l$  instances of an algebraic circuit  $D$  over  $\mathbb{K}$  with  $v$  inputs, in the following sense: given committed values  $x_{i,j}$  and  $z_i$ , with  $i=1, \dots, l$  and  $j=1, \dots, v$ , the prover shows that  $D(x_{i,1}, \dots, x_{i,v}) = z_i$  for  $i=1, \dots, l$ . For circuits with small multiplicative depth, this approach is better than using our first protocol: in fact, the amortized cost may be asymptotically smaller than the number of multiplications in  $D$ .

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**Date:** received 6 Jun 2011

**Contact author:** cramer at cwi nl, ivan@cs au dk, vpastro@cs au dk

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