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## Fully Homomorphic Encryption without Bootstrapping

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**Abstract:** We present a radically new approach to fully homomorphic encryption (FHE) that dramatically improves performance and bases security on weaker assumptions. A central conceptual contribution in our work is a new way of constructing leveled fully homomorphic encryption schemes (capable of evaluating arbitrary polynomial-size circuits), {\em without Gentry's bootstrapping procedure}.

Specifically, we offer a choice of FHE schemes based on the learning with error (LWE) or ring-LWE (RLWE) problems that have  $2^{\secparam}$  security against known attacks. For RLWE, we have:

1. A leveled FHE scheme that can evaluate  $L$ -level arithmetic circuits with  $\tilde{O}(\secparam \cdot L^3)$  per-gate computation -- i.e., computation {\em quasi-linear} in the security parameter. Security is based on RLWE for an approximation factor exponential in  $L$ . This construction does not use the bootstrapping procedure.
2. A leveled FHE scheme that uses bootstrapping {\em as an optimization}, where the per-gate computation (which includes the bootstrapping procedure) is  $\tilde{O}(\secparam^2)$ , {\em independent of  $L$ }. Security is based on the hardness of RLWE for {\em quasi-polynomial} factors (as opposed to the sub-exponential factors needed in previous schemes).

We obtain similar results for LWE, but with worse performance. We introduce a number of further optimizations to our schemes. As an example, for circuits of large width -- e.g., where a constant fraction of levels have width at least  $\secparam$  -- we can reduce the per-gate computation of the bootstrapped version to  $\tilde{O}(\secparam)$ , independent of  $L$ , by {\em batching the bootstrapping operation}. Previous FHE schemes all required  $\tilde{O}(\secparam^{3.5})$  computation per gate.

At the core of our construction is a much more effective approach for managing the noise level of lattice-based ciphertexts as homomorphic operations are performed, using some new techniques recently introduced by Brakerski and Vaikuntanathan (FOCS 2011).

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