On Shor's Factoring Algorithm with More Registers and the Problem to Certify Quantum Computers

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Abstract. Shor's factoring algorithm uses two quantum registers. By introducing more registers we show that the measured numbers in these registers which are of the same pre-measurement state, should be equal if the original Shor's complexity argument is sound. This contradicts the argument that the second register has r possible measured values. There is an anonymous comment which argues that the states in these registers are entangled. If so, the entanglement involving many quantum registers can not be interpreted by the mechanism of EPR pairs and the like. In view of this peculiar entanglement has not yet been mentioned and investigated, we think the claim that the Shor's algorithm runs in polynomial time needs more physical verifications. We also discuss the problem to certify quantum computers.

Keywords. Shor's factoring algorithm, quantum register, entanglement, joint probability, conditional probability.

1 Introduction

It is well-known that factoring an integer n can be reduced to finding the order of an integer x with respect to the module n (G. Miller [1]), which is usually denoted by $\mathrm{ord}_n(x)$. So far, there is not a polynomial time algorithm run on classical computers which can be used to compute $\mathrm{ord}_n(x)$. In 1994, P. Shor [2] proposed a quantum algorithm which is claimed to be possible to compute $\mathrm{ord}_n(x)$ in polynomial time.

The Shor's algorithm uses two quantum registers. It needs an efficient quantum modular exponentiation method. Recently we [3] have found that the Shor's algorithm has to invoke the unitary operation U with $O(q^2)$ times, which can not be implemented in polynomial time, where $n^2 \le q < 2n^2$, n is the large number to be factored, $U|y\rangle \equiv |xy (\mathrm{mod}\, n)\rangle$, $y \in \{0,1\}^\ell$, ℓ is the bit

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length of n. So far, there are few literatures to investigate the mysterious process

$$\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}|a\rangle|0\rangle \rightarrow \frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}|a\rangle|x^a(\operatorname{mod} n)\rangle.$$

We have reported the flaw to some researchers, but only received a comment made by MIT professor Scott Aaronson. He explained that (personal communication, 2014/09/02):

> The repeated squaring algorithm works (and works in polynomial time) for any single $|a\rangle|0\rangle$, mapping it to $|a\rangle|x^a\,(\operatorname{mod} n)\rangle$. But, because of the linearity of quantum mechanics, this immediately implies that the algorithm must also work for any superposition of $|a\rangle$'s, mapping $\sum_a |a\rangle$ to $\sum_a |a\rangle|x^a\,(\operatorname{mod} n)\rangle$.

We do not think that his answer is convincing, because it is too vague to specify *how many and what quantum gates or unitary operations are used on each qubit or a group of qubits in the second quantum register* (see Ref.[3]). Except the the above flaw, Shor's algorithm relates to a peculiar entanglement which has not yet been mentioned and investigated.

At the end of the Shor's factoring algorithm, one should observe the first register and denote the measured result as an integer c. Its complexity argument comprises:

(1) The probability p of seeing a quantum state $|c, x^k\,(\operatorname{mod} n)\rangle$ such that $r/2 \geq \{rc\}_q$ is greater than $1/3r^2$, where n is the integer to be factored, q is a power of 2 satisfying $n^2 \leq q < 2n^2$ and $r = \operatorname{ord}_n(x)$. For convenience, the notation $\operatorname{mod} n$ will be omitted henceforth.

(2) There are $\phi(r)$ possible c which can be used to compute the order r.

(3) The measured number in the second register, i.e., x^k, takes r possible values $1, x, x^2, \cdots, x^{r-1}$.

(4) The success probability of running the algorithm once is greater than $r \cdot \phi(r) \cdot \frac{1}{3r^2}$. Since $\phi(r)/r > \xi/\log\log r$ for some constant ξ, it concludes that the algorithm runs in polynomial time.

Notice that the complexity argument views p as the joint probability $\Pr(X = c, Y = x^k)$, instead of the conditional probability $\Pr(X = c \,|\, Y = x^k)$, where two random variables X and Y assume respectively values from the sets $\{0, 1, \cdots, q-1\}$ and $\{1, x, \cdots, x^{r-1}\}$.

In the extended abstract of this paper, we [4] introduced a more quantum register into Shor's algorithm. On the one hand, by Shor's argument we proved that

$$\Pr(X = c, Y = x^k, Z = x^k) = \Pr(X = c, Y = x^k).$$

On the other hand, if the observed values in the latter two quantum registers are random, we have

$$\Pr(X = c, Y = x^k, Z = x^k) < \Pr(X = c, Y = x^k, Z = x^l), \quad \text{where } k \neq l.$$

But this contradicts that

$$\begin{aligned}\Pr(X = c, Y = x^k) &\geq \Pr(X = c, Y = x^k) \cdot \Pr(Z = x^l | \{X = c, Y = x^k\}) \\ &= \Pr(X = c, Y = x^k, Z = x^l)\end{aligned}$$

where $\Pr(Z = x^l | \{X = c, Y = x^k\})$ is the conditional probability. It shows that the measured numbers in two quantum registers of the same pre-measurement state should be equal if Shor's complexity argument is sound. However, this contradicts the argument that there are r different observed values for the second register. So far, we have only received one anonymous comment on the extended abstract [4]. It argues that the states in the three quantum registers are entangled.

In this paper, we shall introduce more quantum registers into Shor's algorithm. We show that the measured numbers in the added quantum registers should still be equal to the measured number in the second register if Shor's factoring algorithm runs in polynomial time. That is to say, all these quantum qubits in these registers should be entangled by the above comment. But we find that the entanglement involving many quantum registers can not be interpreted by the mechanism of EPR pairs and the like. Moreover, this is a peculiar entanglement which has never been mentioned and investigated. Since the complexity of Shor's factoring algorithm depends essentially on the peculiar entanglement, we think the claim that Shor's algorithm runs in polynomial time is not doubtless from a theoretical point of view.

2 Preliminary

A quantum analogue of a classical computer operates with quantum bits involving quantum states. The state of a quantum computer is described as a basis vector in a Hilbert space. A qubit is a quantum state $|\Psi\rangle$ of the form

$$|\Psi\rangle = a|0\rangle + b|1\rangle,$$

where the amplitudes $a, b \in \mathbb{C}$ such that $|a|^2 + |b|^2 = 1$, $|0\rangle$ and $|1\rangle$ are basis vectors of the Hilbert space. Here, the *ket* notation $|x\rangle$ means that x is a quantum state. The state of a quantum system having n qubits is a point in a 2^n-dimensional vector space. Given a state

$$\sum_{i=0}^{2^n-1} a_i|\chi_i\rangle,$$

where the amplitudes are complex numbers such that $\sum_{i=0}^{2^n-1} |a_i|^2 = 1$ and each $|\chi_i\rangle$ is a basis vector of the Hilbert space, if the machine is measured with respect to this basis, the probability of seeing basis state $|\chi_i\rangle$ is $|a_i|^2$.

Two quantum mechanical systems are combined using the tensor product. For example, a system of two qubits $|\Psi\rangle = a_1|0\rangle + a_2|1\rangle$ and $|\Phi\rangle = b_1|0\rangle + b_2|1\rangle$ can be written as

$$|\Psi\rangle|\Phi\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 b_1 \\ a_1 b_2 \\ a_2 b_1 \\ a_2 b_2 \end{pmatrix}$$

We shall also use the shorthand notations $|\Psi, \Phi\rangle$. We call a quantum state having two or more components *entangled* state, if it is not a product state. According to the Copenhagen interpretation of quantum mechanics, measurement causes an instantaneous collapse of the wave function describing the quantum system into an eigenstate of the observable state that was measured. If entangled, one object cannot be fully described without considering the other(s).

The entangled state

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

is commonly referred to as an EPR pair, after Einstein, Podolsky and Rosen who tried to use it to prove Quantum Mechanics incomplete. Suppose that the first qubit of the state above is given to Alice and the second one is given to Bob. When Alice measures her qubit in the computational basis, she will obtain the outcomes $|0\rangle$ or $|1\rangle$ with equal probability. Thus, the system shall collapse to either $|01\rangle$ or $|10\rangle$ because the two qubits are entangled. This means that, if Bob measures his qubit afterwards, his outcome is completely determined by Alice's measurement result.

In short, EPR pair is a form of quantum superposition. When a measurement is made and it causes one member of such a pair to take on a definite value, the other member of this entangled pair will at any subsequent time be found to have taken the appropriately correlated value. Thus, there is a correlation between the results of measurements performed on entangled pairs, and this correlation is observed even though the entangled pair may have been separated by arbitrarily large distances.

3 Description of Shor's factoring algorithm

Shor's factoring algorithm proceeds as follows [2]. At the beginning of the algorithm, one has to find $q = 2^s$ for some integer s such that $n^2 \le q < 2n^2$, where n is to be factored.

Initialization. Put register-1 in the following uniform superposition

$$\frac{1}{\sqrt{q}} \sum_{a=0}^{q-1} |a\rangle|0\rangle.$$

Computation. Keep a in register-1 and compute x^a in register-2 for some randomly chosen integer x. We then have the following state

$$\frac{1}{\sqrt{q}} \sum_{a=0}^{q-1} |a\rangle|x^a\rangle.$$

Fourier transformation. Performing Fourier transform on register-1 [5, 6], we obtain the following state

$$\frac{1}{q}\sum_{a=0}^{q-1}\sum_{c=0}^{q-1}\exp(2\pi iac/q)|c\rangle|x^a\rangle.$$

Observation. It suffices to observe the first register. The probability p that the machine reaches the state $|c, x^k\rangle$ is

$$\left|\frac{1}{q}\sum_{a:\, x^a\equiv x^k}\exp(2\pi iac/q)\right|^2 \tag{1}$$

where $0 \leq k < r = \mathrm{ord}_n(x)$, the sum is over all $a\,(0 \leq a < q)$ such that $x^a \equiv x^k$.

Continued fraction expansion. If there is a d such that

$$\frac{-r}{2} \leq dq - rc \leq \frac{r}{2}$$

then the probability of seeing $|c, x^k\rangle$ is greater than $1/3r^2$ (see the following section for the argument of this claim). Hence, we have

$$\left|\frac{d}{r} - \frac{c}{q}\right| \leq \frac{1}{2q}$$

Since $q \geq n^2$, we can round c/q to obtain d/r. Thus r can be obtained.

4 The complexity argument of Shor's factoring algorithm

Here is a brief description of the complexity argument of Shor's factoring algorithm. We refer to Ref.[2] for details. Setting $a = br + k$ for some integer b and the order $r = \mathrm{ord}_n(x)$, the probability p is

$$\left|\frac{1}{q}\sum_{b=0}^{\lfloor (q-k-1)/r\rfloor} e^{2\pi i(br+k)c/q}\right|^2.$$

Then it argues that the probability p equals to

$$\left|\frac{1}{q}\sum_{b=0}^{\lfloor (q-k-1)/r\rfloor} e^{2\pi ib\{rc\}_q/q}\right|^2.$$

Writing the above sum into an integral, we obtain

$$\frac{1}{q}\int_{b=0}^{\lfloor\frac{(q-k-1)}{r}\rfloor} e^{2\pi ib\{rc\}_q/q}\mathrm{d}b + O\left(\frac{\lfloor (q-k-1)/r\rfloor}{q}(e^{2\pi i\{rc\}_q/q}-1)\right).$$

Taking $u = rb/q$, we have

$$\frac{1}{r}\int_{0}^{\frac{r}{q}\lfloor\frac{q-k-1}{r}\rfloor}\exp\left(2\pi iu\frac{\{rc\}_q}{r}\right)\mathrm{d}u$$

Taking into account that $k < r$, we can obtain the approximation

$$\frac{1}{r}\int_0^1 \exp\left(2\pi i u \frac{\{rc\}_q}{r}\right) \mathrm{d}u.$$

Hence, we have

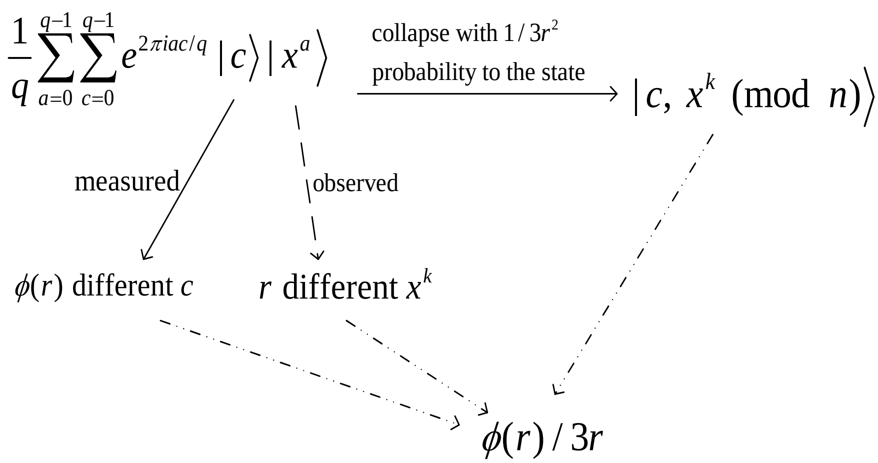
$$\left|\frac{1}{r}\int_0^1 \exp\left(2\pi i u \frac{\{rc\}_q}{r}\right) \mathrm{d}u\right| \geq 2/(\pi r)$$

Therefore,

$$p \geq \frac{4}{\pi^2 r^2} > 1/3r^2.$$

Since $r = \mathrm{ord}_n(x)$, there are r different $x^k \mod n$. If $(d, r) = 1$, there are $\phi(r)$ different d where ϕ is Euler's totient function [7, 8]. Thus, the probability of obtaining r is greater than $\phi(r) \cdot r \cdot 1/3r^2 = \phi(r)/3r$. This leads to a polynomial time algorithm for factorization.

For convenience, we now depict the Shor's complexity argument by Graph-1.



Graph-1: Shor's complexity argument

5 On the lower bound to the joint probability

It is easy to find that P. Shor views $1/3r^2$ as the lower bound to the joint probability $P(X = c, Y = x^k)$, where $r/2 \geq \{rc\}_q$, the random variables X and Y values respectively from the sets $\{0, 1, \cdots, q-1\}$ and $\{1, x, \cdots, x^{r-1}\}$. Frankly speaking, it is difficult to argue mathematically whether the expression of Eq.(1) is a joint probability or not. *But we here stress that the expression of Eq.(1) is directly defined over x^k which is the observed value in the second register.* We shall argue that it is better to view p as the conditional probability $P(X = c \,|\, Y = x^k)$ rather than the joint probability $P(X = c, Y = x^k)$. The basic idea behind our argument is to investigate a variation of Shor's factoring algorithm which requires three quantum registers.

Given an integer n which is to be factored, find $q = 2^s$ for some integer s such that $n^2 \leq q < 2n^2$.

[Initialization] Put register-1 in the following uniform superposition state

$$\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}|a\rangle|0|0\rangle.$$

[Computation] Keep a in register-1 and compute x^a in register-2 and register-3 for some randomly chosen integer x, $1 < x < n$. The state becomes

$$\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}|a\rangle|x^a|x^a\rangle.$$

[Fourier transformation] Perform Fourier transform on register-1. The state becomes

$$\frac{1}{q}\sum_{a=0}^{q-1}\sum_{c=0}^{q-1}\exp(2\pi iac/q)|c\rangle|x^a|x^a\rangle.$$

[Observation] Observe register-1. The probability p that the machine ends in the state $|c, x^k, x^k\rangle$ is

$$\left|\frac{1}{q}\sum_{a\colon x^a\equiv x^k}\exp(2\pi iac/q)\right|^2 \tag{1'}$$

where $0 \le k < r = \mathrm{ord}_n(x)$, the sum is over all $a\,(0 \le a < q)$ such that $x^a \equiv x^k$.

To see the similarities of Shor's algorithm with two registers and the variation with three registers, we refer to the following Table-1.

Shor's algorithm with two registers	A variation of Shor's algorithm with three registers
(I) *Put* register-1 in the uniform superposition. The state becomes $\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}\|a\rangle\|0\rangle$	(I) *Put* register-1 in the uniform superposition. The state becomes $\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}\|a\rangle\|0\rangle\|0\rangle$
(II) *Compute* $x^a \pmod n$ in register-2. The state becomes $\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}\|a\rangle\|x^a\rangle$	(II) *Compute* $x^a \pmod n$ in register-2. and register-3. The state becomes $\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}\|a\rangle\|x^a\rangle\|x^a\rangle$
(III) *Perform* Fourier transform on register-1. The state becomes $\frac{1}{q}\sum_{a=0}^{q-1}\sum_{c=0}^{q-1}e^{2\pi iac/q}\|c\rangle\|x^a\rangle$	(III) *Perform* Fourier transform on register-1. The state becomes $\frac{1}{q}\sum_{a=0}^{q-1}\sum_{c=0}^{q-1}e^{2\pi iac/q}\|c\rangle\|x^a\rangle\|x^a\rangle$
(IV) *Observe* the machine and *compute* the probability of seeing the state $\|c, x^k\rangle$. It is $\left\|\frac{1}{q}\sum_{a\colon x^a\equiv x^k}e^{2\pi iac/q}\right\|^2$	(IV) *Observe* the machine and *compute* the probability of seeing the state $\|c, x^k, x^k\rangle$. It is $\left\|\frac{1}{q}\sum_{a\colon x^a\equiv x^k}e^{2\pi iac/q}\right\|^2$

Table-1: Similarities of Shor's factoring algorithm and its variation with three registers

Since the Eq.(1) is the same as Eq.(1'), by Shor's argument we easily prove that

$$\Pr\left(X=c, Y=x^k\right)=\Pr\left(X=c, Y=x^k, Z=x^k\right) \tag{2}$$

However, if the measured results in register-2 and register-3 are *random*, then

$$\Pr\left(X=c, Y=x^k, Z=x^k\right)<\Pr\left(X=c, Y=x^k, Z=x^l\right), \tag{3}$$

where $0 \leq k, l \leq r-1, k \neq l$. Thus,

$$\begin{aligned}
\Pr\left(X=c, Y=x^k\right) &\geq \Pr\left(X=c, Y=x^k\right) \cdot \Pr\left(Z=x^l \mid \{X=c, Y=x^k\}\right) \\
&= \Pr\left(X=c, Y=x^k, Z=x^l\right).
\end{aligned}$$

This is a contradiction. Notice that the contradiction originates directly from the Eq.(2).

So far, there are only two suggestions to resolve the above contradiction:

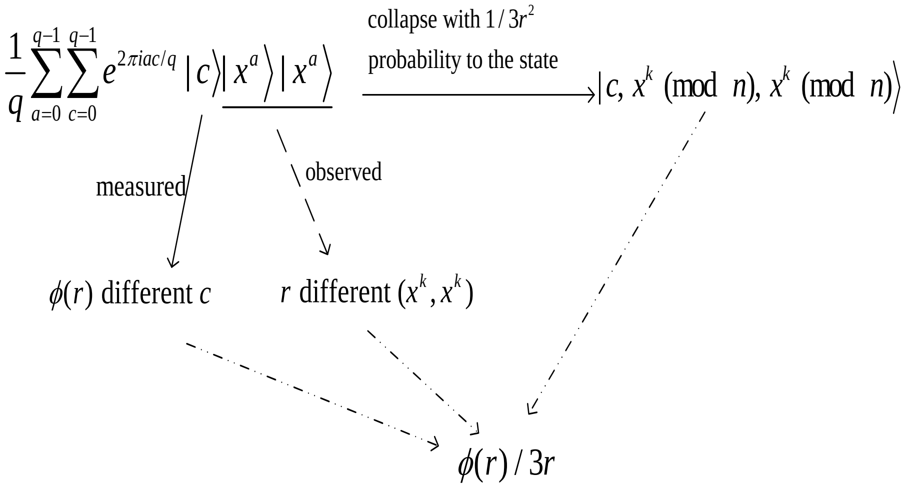
(I) In our opinion, the expression $\left|\frac{1}{q} \sum_{a:\, x^a \equiv x^k} \exp(2\pi iac/q)\right|^2$ should be viewed as the conditional probability $\Pr\left(X=c \mid Y=x^k\right)$, not the joint probability $\Pr\left(X=c, Y=x^k\right)$. In such case, we can not obtain Eq.(2). Moreover, the success probability of running Shor's factoring algorithm once is greater than $\phi(r)/3r^2$, not $\phi(r)/3r$. Thus, the claim that Shor's factoring algorithm takes polynomial time is flawed.

(II) The other suggestion claims that the machine always ends in the state $|c, x^k, x^k\rangle$. It argues that all qubits in three quantum registers are entangled, and insists that the state $|c, x^k, x^l\rangle$ where $k \neq l$ can not be observed.

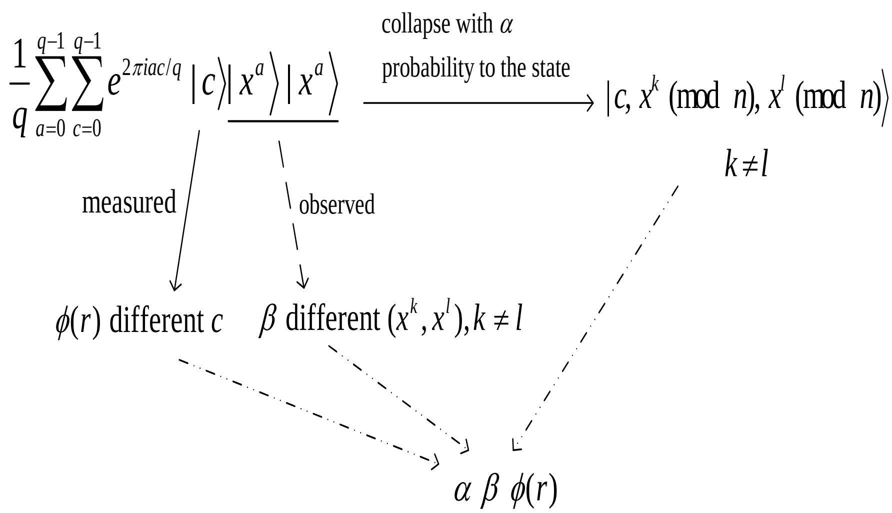
6 Does the machine always end in the state $|c, x^k, x^k\rangle$

There is a comment on the manuscript. It argues that: *if the second register is exactly the same as the third register, then the probability of measuring two different numbers in the second and third registers is indeed zero.* That is to say, the state $|c, x^k, x^l\rangle, k \neq l$ could not be observed if the latter two quantum registers have the same pre-measurement state. The machine always ends in the state $|c, x^k, x^k\rangle$. The argument is depicted by Graph-2. This argument insists that all qubits in the three quantum registers are entangled. It claims that in Graph-3 the probability α should be zero.

By the argument, if we introduce more quantum registers, we shall find that the measured numbers in the added quantum registers should still be equal to the the measured number in the second register(see Table-2). In other words, the argument claims that all qubits in those quantum registers should be entangled.



Graph-2: The argument that the machine always ends in the state $|c, x^k, x^k\rangle$



Graph-3: The probability that the machine ends in the state $|c, x^k, x^l\rangle, k \neq l$

If the peculiar entanglement is indispensable for Shor's factoring algorithm, we should point out that:

(1) The entanglement involving many quantum registers has not yet been mentioned and investigated. All literatures related to Shor's factoring algorithm, such as Ref.[9, 10, 11], did not consider the peculiar entanglement. In 2006, V. Kendon and W. Munro [10] investigated the topic that entanglement and its role in Shor's algorithm. They point out that:

> *The nature of entanglement generated in the process of running Shor's factoring algorithm is still not fully understood, and little has been said about what role entanglement actually plays in quantum computation.*

They [10] used "numerical simulation" to investigate how entanglement between register qubits varies as Shor's algorithm is run on a quantum computer. They concluded that the inverse quantum Fourier transform can only generate entanglement within the upper register, or, move entanglement around between the upper register qubits.

(2) It seems impossible to interpret the event (the measured numbers in two quantum registers with the same pre-measurement state should be equal) by the mechanism of EPR pairs and the like. Intuitively, the phenomenon violates Heisenberg's uncertainty principle. If the peculiar entanglement is indeed present during the course of Shor's algorithm, how to define the entanglement and how to certify it?

(3) Surprisingly, the term "entanglement" does not appear in Shor's paper [2]. It seems that the inventor did not realize at that time that his complexity argument of the famous algorithm must rely on the peculiar entanglement.

In summary, the claim that the machine always ends in the state,

$$|c, \underbrace{x^k, \cdots, x^k}_{\ell}\rangle$$

in our opinion, is not convincing because it directly contradicts Shor's argument that there are r different observed values for register-2.

Shor's algorithm with two registers	A variation of Shor's algorithm with $\ell+1$ registers
(I) *Put* register-1 in the uniform superposition. The state becomes $\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}\|a\rangle\|0\rangle$	(I) *Put* register-1 in the uniform superposition. The state becomes $\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}\|a\rangle\underbrace{\|0\rangle\cdots\|0\rangle}_{\ell}$
(II) *Compute* x^a in register-2. The state is $\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}\|a\rangle\|x^a\rangle$	(II) *Compute* x^a in the latter ℓ registers. The state is $\frac{1}{\sqrt{q}}\sum_{a=0}^{q-1}\|a\rangle\underbrace{\|x^a\rangle\cdots\|x^a\rangle}_{\ell}$
(III) *Perform* Fourier transform on register-1. The state becomes $\frac{1}{q}\sum_{a=0}^{q-1}\sum_{c=0}^{q-1}e^{2\pi iac/q}\|c\rangle\|x^a\rangle$	(III) *Perform* Fourier transform on register-1. The state becomes $\frac{1}{q}\sum_{a=0}^{q-1}\sum_{c=0}^{q-1}e^{2\pi iac/q}\|c\rangle\underbrace{\|x^a\rangle\cdots\|x^a\rangle}_{\ell}$
(IV) *Observe* the machine and *compute* the probability of seeing the state $\|x, x^k\rangle$. It is $\left\|\frac{1}{q}\sum_{a:\, x^a\equiv x^k}e^{2\pi iac/q}\right\|^2$	(IV) *Observe* the machine and *compute* the probability of seeing the state $\|c, \underbrace{x^k, \cdots x^k}_{\ell}\rangle$. It is $\left\|\frac{1}{q}\sum_{a:\, x^a\equiv x^k}e^{2\pi iac/q}\right\|^2$

Table-2: A variation of Shor's algorithm with $\ell+1$ registers

7 The latest experiments on entanglement

In 2012, it [12] reported a new experiment on generating, manipulating and measuring entanglement. In the experiment *four photons* are used. "It can delay the choice of measurement-implemented through a high-speed tunable bipartite-state analyser and a quantum random-number generator on two of the photons into the time-like future of the registration of the other two photons. This effectively projects the two already registered photons onto one of two mutually exclusive quantum states in which the photons are either entangled (quantum correlations) or separable (classical correlations)."

In another experiment, P. Shadbolt, et al. [13] reported an integrated waveguide device that can generate and completely characterize pure *two-photon* states with any amount of entanglement and arbitrary single-photon states with any amount of mixture.

8 Using Shor's factoring algorithm to certify a quantum computer

In 2001, Shor's algorithm was demonstrated by a group at IBM, who factored 15 into 3×5. However, some doubts that whether IBM's experiment was a true demonstration of quantum computation have been raised, since no entanglement was observed.

In 2012, E. Lucero et al. [14] claimed that they have run a three-qubit compiled version of Shor's algorithm to factor the number 15. They produce coherent interactions between five qubits and verify bi- and tripartite entanglement via quantum state tomography.

In the past ten years, it is somewhat depressing that several demonstrations of Shor's factoring algorithm only claimed to be able to factor 15 or 21. It is even worse that almost these demonstrations are criticised for lack of more scrutiny.

On May 11, 2011, D-Wave Systems [15] announced the D-Wave One, labeled "the world's first commercially available quantum computer." The company claims this system uses a 128 qubit processor chipset. In early 2012 D-Wave Systems revealed a 512-qubit quantum computer. In January 2014, researchers at UC Berkeley and IBM published a classical model explaining the D-Wave machine's observed behavior, suggesting that it may not be a quantum computer [16]. In May 2014, researchers at D-Wave, Google, USC, Simon Fraser University, and National Research Tomsk Polytechnic University published a paper containing experimental results that demonstrated the presence of entanglement among D-Wave qubits [17].

Historically speaking, Shor's factoring algorithm is the main impetus of developing quantum computers. In some senses, a computer could be called a "quantum computer" only if it is able to factorize large integers by running Shor's factoring algorithm. We believe it is a universally acceptable standard to certify a quantum computer using Shor's factoring algorithm. But it is a pity, D-Wave has no intention to use its quantum computers to run the famous algorithm so as to certify itself.

9 Conclusion

In this paper, by investigating Shor's factoring algorithm with more registers, we show that the measured numbers in many quantum registers with the same pre-measurement state should be equal if the original argument is sound. We think the claim that Shor's factoring algorithm runs in polynomial time needs more investigations, especially, physical verifications for the peculiar phenomenon. We also throw some light on the question of what a quantum computer was like.

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