Large Deviations for the Empirical Mean of an M/M/1 Queue

J. Blanchet , P. W. Glynn, and S. Meyn

Queueing Systems: Theory and Applications, Vol 73, No. 4, 425-446 (2013)

- BlanchetGMeyn13.pdf

Let $(Q(k) : k \geq 0)$ be an M/M/1 queue with traffic intensity $\rho \in (0, 1)$. Consider the quantity

$$S_n(p) = \frac{1}{n} \sum_{j=1}^{n} Q(j)^p$$

for any $p > 0$. The ergodic theorem yields that $S_n(p) \to \mu(p) := E[Q(\infty)p]$, where $Q(\infty)$ is geometrically distributed with mean $\rho/(1-\rho)$. It is known that one can explicitly characterize $I(\epsilon) > 0$ such that

$$\lim_{n\to\infty} \frac{1}{n} \log P\big(S_n(p) < \mu(p) - \epsilon\big) = -I(\epsilon), \quad \epsilon > 0$$

In this paper, we show that the approximation of the right tail asymp-totics requires a different logarithm scaling, giving

$$\lim_{n\to\infty} \frac{1}{n^{1/(1+p)}} \log P(S_n(p) > \mu(p) + \epsilon) = -C(p)\epsilon^{1/(1+p)},$$

where $C(p) > 0$ is obtained as the solution of a variational problem.

We discuss why this phenomenon — Weibullian right tail asymptotics rather than exponential asymptotics — can be expected to occur in more general queueing systems.

Page generated 2013-08-11 18:52:55 PDT, by jemdoc.