

# Uniform Renewal Theory with Applications to Expansions of Random Geometric Sums

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*J. Blanchet and P. W. Glynn*

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Consider a sequence  $X = (X_n; n \geq 1)$  of independent and identically distributed random variables, and an independent geometrically distributed random variable  $M$  with parameter  $p$ . The random variable  $S_M = X_1 + \dots + X_M$  is called a geometric sum. In this paper we obtain asymptotic expansions for the distribution of  $S_M$  as  $p \rightarrow 0$ . If  $EX_1 > 0$ , the asymptotic expansion is developed in powers of  $p$  and it provides higher-order correction terms to Renyi's theorem, which states that  $P(pS_M > x) \approx \exp(-x/EX_1)$ . Conversely, if  $EX_1 = 0$ , then the expansion is given in powers of  $\sqrt{p}$ . We apply the results to obtain corrected diffusion approximations for the  $M/G/1$  queue. These expansions follow in a unified way as a consequence of new uniform renewal theory results that are also developed in this paper.

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