Mixing Times for Random Walks on Geometric Random Graphs

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A geometric random graph, $G^d(n,r)$, is formed as follows: place n nodes uniformly at random onto the surface of the d-dimensional unit torus and connect nodes which are within a distance r of each other. The $G^d(n,r)$ has been of great interest due to its success as a model for ad-hoc wireless networks. It is well known that the connectivity of $G^d(n,r)$ exhibits a threshold property: there exists a constant α_d such that for any $\epsilon > 0$, for $r^d < \alpha_d(1-\epsilon)\log n/n$, the $G^d(n,r)$ is not connected with high probability, and for $r^d > \alpha_d(1+\epsilon)\log n/n$, the $G^d(n,r)$ is connected w.h.p. In this paper, we study mixing properties of random walks on $G^d(n,r)$ for $r^d(n) = \Omega(\log n/n)$. Specifically, we study the scaling of mixing times of the fastest-mixing reversible random walk, and the natural random walk. We find that the mixing time of both of these random walks have the same scaling laws and scale proportional to r^{-2} (for all d). These results hold for $G^d(n,r)$ when distance is defined using any L_p norm. Though the results of this paper are not so surprising, they are nontrivial and require new methods. To obtain the scaling law for the fastest-mixing reversible random walk, we first explicitly characterize the fastest-mixing reversible random walk on a regular (grid-type) graph in d dimensions. We subsequently use this to bound the mixing time of the fastest-mixing random walk on $G^d(n,r)$. In the course of our analysis, we obtain a tight relation between the mixing time of the fastest-mixing symmetric random walk and the fastest-mixing reversible random walk with a specified equilibrium distribution on an arbitrary graph. To study the natural random walk, we first generalize a method of Diaconis and Stroock (1991) to bound eigenvalues based on Poincare's inequality and then apply it to the $G^d(n,r)$ graph.
