Heavy-Traffic Extreme-Value Limits for Queues

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We consider the maximum waiting time among the first n customers in the GI/G/1 queue. We use strong approximations to prove, under regularity conditions, convergence of the normalized maximum wait to the Gumbel extreme-value distribution when the traffic intensity ρ approaches 1 from below and n approaches infinity at a suitable rate. The normalization depends on the interarrival-time and service-time distributions only through their first two moments, corresponding to the iterated limit in which first ρ approaches 1 and then n approaches infinity. We need n to approach infinity sufficiently fast so that $n(1-\rho)^2 \to \infty$. We also need n to approach infinity sufficiently slowly: If the service time has a p^{th} moment for ρ>2, then it suffices for $(1-\rho)n^{1/p}$ to remain bounded; if the service time has a finite moment generating function, then it suffices to have $(1-\rho)\log n \to 0$. This limit can hold even when the normalized maximum waiting time fails to converge to the Gumbel distribution as $n \to \infty$ for each fixed ρ. Similar limits hold for the queue length process.