Robust Minimum Variance Beamforming

R. Lorenz and S. Boyd

IEEE Transactions on Signal Processing, 53(5), p1684-1696, May 2005.
A longer version appeared as chapter 1 of *Robust Adaptive Beamforming*, edited by P. Stoica and J. Li, Wiley, 2006, pages 1-47, ISBN 0-471-67850-3.
A shorter version appears in *Proceedings Asilomar Conference on Signals, Systems and Computers*, 2:1345-1352, November 2003.

- rmvb.pdf
- rmvbBookChapter.pdf
- rmvb_asilomar.pdf

This paper introduces an extension of minimum variance beamforming, also known as Capon's method, that explicitly takes into account variation or uncertainty in the assumed array response. Sources of this uncertainty include imprecise knowledge of the angle of arrival and uncertainty in the array manifold. In Capon's method, the weights are chosen to minimize the weighted array power output subject to a unity gain constraint in the desired look direction. This method assumes that the array manifold is precisely known; unfortunately, even small variations in the array manifold can drastically reduce its performance. In our method, uncertainty in the array manifold is explicitly modeled via an emph{uncertainty ellipsoid} that gives the possible values of the array for a particular look direction. We choose weights that minimize the total weighted power output of the array, subject to the constraint that the gain should exceed unity for all array responses in this uncertainty ellipsoid. If the ellipsoid reduces to a single point, the method coincides with Capon's method. Unlike Capon's method, however, we can guarantee performance of the robust method in the presence of uncertainties. We show that the robust weights can be computed efficiently using Lagrange multiplier techniques. In fact, the robust weight selection problem can be solved in the same order of complexity as the non-robust counterpart. We describe in detail several methods that can be used to derive an appropriate uncertainty ellipsoid for the array response. The simplest methods fit an ellipsoid around empirical (or simulated) data. In a more sophisticated approach, we form separate uncertainty ellipsoids for each component in the signal path (e.g., antenna, electronics) and then determine an aggregate uncertainty ellipsoid from these. We give new results for modeling the element-wise products of ellipsoids. We demonstrate the robust beamforming and the ellipsoidal modeling methods with several numerical examples.