Strongly Efficent Algorithms for Light-tailed Random Walks: An Old Folk Song Sung to a Faster New Tune

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We revisit a classical problem in rare-event simulation, namely, efficient estimation of the probability that the sample mean of n independent identically distributed light tailed (i.e. with finite moment generating function in a neighborhood of the origin) random variables lies in a sufficiently regular closed convex set that does not contain their mean. It is well known that the optimal exponential tilting (OET), although logarithmically efficient, is not strongly efficient (typically, the squared coefficient of variation of the estimator grows at rate $n^{1/2}$). After discussing some important differences between the optimal change of measure and OET (for instance, in the one dimensional case the size of the overshoot is bounded for the optimal importance sampler and of order $O(n^{1/2})$ for OET) that indicate why OET is not strongly efficient, we provide a state-dependent importance sampling that can be proved to be strongly efficient. Our procedure is obtained based on computing the optimal tilting at each step, which corresponds to the solution of the Isaacs equation studied recently by Dupuis and Wang (2004)*.

*Dupuis, P., Wang, H.: Importance sampling, large deviations, and differential games. Stoch. and Stoch. Reports 76, 481–508 (2004)