Practice Article

Golf course revenue management

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ABSTRACT This research is based on an analysis of golf course tee-time reservation practice. Specifically, this article presents a unique linear model that can be used to assign the demand to the available tee-times, and thus, maximize their utilization and the total revenue. The model is solved by using the SAS-OR built-in branch and bound (B&B) algorithm. To reduce the computational time, we propose a heuristic to find an initial feasible solution to the model. This initial solution reduces the CPU time substantially and enabled us to solve the larger-scale problem by using the SAS-OR.

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INTRODUCTION

Today, many industries including airlines, car rentals, hotel chains and golf courses consider the revenue management (RM) system as a critical determinant of their future success. The history of (RM) goes back to the major airlines in the United States in the late 1980s; for more information about the origin of this system, see Belobaba (1987) and Talluri and Van Ryzin (2005).

Golf course tee-time assignment is a typical RM problem. Golf course operations are identified with perishable inventory, predictable demand, limited capacity and varying costumer price sensitivity (Kimes and Schruben, 2002). With respect to this revenue problem, a set of tee-times are predetermined for each golf course. The maximum capacity for each tee-time is four, and the number of demand for each tee-time can be forecasted. Each demand type has a potential revenue for the golf course. Demand is assumed to come from groups or individuals, the size of which ranges from 1 to 4. The objective in solving this problem is to assign the demand to tee-times so to maximize the total revenue.

The goal of the golf course RM system is to maximize the profits by developing the best reservation policy. In this article, we assume such a reservation policy, for which the value or bid price of a particular tee-time is periodically evaluated. Each time a new reservation occurs, the algorithm is applied once assuming the demand forecast is made ready at that moment. The reservation is accepted if the expected revenue from this reservation is at least as high as the predetermined bid price. The algorithm we introduce is based on a linear deterministic model that balances the supply and demand. This linear model is repeatedly applied over time, and provides an update of bid prices as demand is being realized. This model is a typical RM method used in practice by the airlines and other industries, the essence of which is to compare the future expected revenue with a current offered price. Therefore, the fundamental RM decision would be to accept or reject a reservation.

An additional explanation may help to better articulate the problem. Optimization is conducted by using the forecasted demand – the demand for each tee-time interval between 0730 hours and 1500 hours. Three rate categories are available for the tee-times, depending on the time of the day, for example, early morning tee-times are more expensive. The prices in each rate category include discount and regular prices, which are used to differentiate the demand with different profit potentials.

LITERATURE REVIEW

Currently, RM is applied extensively to transportation, hotel, media and hospital management. Generally speaking, the methodologies can be classified into two classes according to dynamic (Feng and Xiao, 2000) versus static (Bertsimas and Popescu, 2003) or single leg (Liang, 1999) versus network of two or more legs (Barnhart *et al*, 2002; Wang and Meng, 2008; Barnhart *et al*, 2009). Usually, static models treat demand deterministically and resort to a repeated application of a static capacity allocation model called 'out rollout policy' (Bertsimas and Popescu, 2003). Optimal models are generally dynamic and dictate the threshold price change whenever the remaining inventory changes or a substantial amount of time elapses. An interesting paper by Gallego and van Ryzin (1997) shows that a repeated application of a simple linear programming (LP) model gives an asymptotically optimal policy on a network. Bertsimas and Popescu (2003) also discuss the performance of repeatedly using the LP model in the airlines' network seat inventory control. We follow a similar process to develop LP models for golf course RM. Particularly, for the first time, our models address the special features of this RM problem in the golf course industry.

Similar approaches to the early practices used for RM in the airline industry were also developed for hotel RM to balance the expected revenue from sold rooms and the cost of 'walking away' customers who fail to honor their reservations (Bitran and Mondschein, 1995; Bitran and Gilbert, 1996). The management of golf course tee-time reservations through assigning a set of sequential reservations, shares similarities with the problem of vehicle routing using time windows. Vehicle routing and scheduling problems with time windows are time-constrained network optimization problems in which a set of trips satisfying demand requirements are assigned to a set of vehicles. The cost of assignment is the total cost on all the routes. The objective is to minimize the assignment cost (Hadjer *et al*, 2006). Similarly, in the golf course assignment problem, a set of sequential reservations – each covering the entire time period of a service day – are assigned to a set of tee-times. Each golf course serves a sequence of reservations in a day. The objective is to maximize the assignment revenue. In this sense, the golf course RM problem shares great similarity with the vehicle routing and scheduling problem. In a general sense, the golf course RM falls into a large class of assignment problems, such as the assignment of jobs to machines. The assignment problem can address complex problems from the traveling salesman problem (Held and Karp, 1970) to vehicle routing problems (Li and Wang, 2005; Li *et al,* 2009).

The remainder of the present article is organized as follows: next section describes the problem formulation for the golf course assignment problem; subsequent section presents the numerical results; and the last section is devoted to our concluding remarks.

PROBLEM FORMULATION

As the demand increases, the golf course management must make decisions about how to optimally allocate their resources for future demand. The Golf Revenue Optimization (GRO) model is an assignment problem to optimally allocate the reservations for tee-times. GRO is a revenue maximization problem that takes into account the business rules and constraints defined by regular golf courses.

In this problem, a set of tee-times is deterministic at each golf course. The maximum capacity for each tee-time is four, and the number of reservations (demands) for each tee-time each day is forecastable. Each demand type has a deterministic potential with respect to the revenue of the golf course. Demand is assumed to come as groups or individuals, the size of which ranges from 1 to 4. The objective in this problem is to assign the reservations to tee-times so to maximize the total revenue.

Here x_{ij} is defined as a binary variable where $x_{ij}=1$ if reservation i is assigned to tee-time j; 0, otherwise. Similarly $z_j=1$ if there is at least one reservation assigned to tee-time j; 0, otherwise. The variable z_j is defined so that the model assigns as many reservations to a tee-time as possible. For example, this strategy forces three parties of size 1 to reserve one tee-time instead of three different tee-times. The variables y_i^+ and y_i^- are integer variables indicating the time deviation from the costumer's requested tee-time T_i. Therefore, the mathematical model for the GRO can be formulated as:

$$[\text{GRO}] \quad \text{Maximize} \sum_{i\in I}\sum_{j\in T} r_{ij}\, x_{ij} - \sum_{j\in T} c_j z_j - \sum_{i\in I} p_i^+ y_i^+ - \sum_{i\in I} p_i^- y_i^-$$

subject to:

$$\sum_{j\in T} x_{ij} \leqslant 1 \quad \forall i \in I \qquad (1)$$

$$\sum_{i\in I} s_i x_{ij} \leqslant 4 \quad \forall j \in T \qquad (2)$$

$$t_j \cdot x_{ij} \geqslant t_{is} \quad \forall i \in I, \quad \forall j \in T \qquad (3)$$

$$t_j \cdot x_{ij} \leqslant t_{ie} \quad \forall i \in I, \quad \forall j \in T \qquad (4)$$

$$z_j \geqslant x_{ij} \quad \forall j \in T \qquad (5)$$

$$t_j \cdot x_{ij} + y_i^+ - y_i^- = T_i \quad \forall i \in I, \quad \forall j \in T \qquad (6)$$

$$x_{ij}, z_j = 0, 1 \quad \forall i \in I, \quad \forall j \in T \qquad (7)$$

$$y_i^+, y_i^- \geqslant 0, \quad \text{Integer} \quad \forall i \in I, \quad \forall j \in T \qquad (8)$$

where

I	is the set of all reservations (Parties)
T	is the set of all tee-time intervals in a day
s_i	is the size of party
r_{ij}	is a revenue associated with booking i assigned to tee-time j
p_i^+ and p_i^-	are the penalties associated with the time deviation from the costumer's requested tee-time
c_j	is the cost associated with tee-time j, if there is any assignment to this tee-time
t_j	is the time at tee-time j
party i	is allowed to be assigned to a tee-time j in a time window between t_{is} and t_{ie}
T_i	is the tee-time requested by a costumer
$x_{ij}=1$	if reservation i is assigned to tee-time j; otherwise, the value $=0$
$z_j=1$	if at least one reservation is assigned to tee-time j; otherwise, the value $=0$
y_i^+	is time deviation up to 1 hour after the customer's requested tee-time
y_i^-	is time deviation up to 1 hour before the customer's requested tee-time

In GRO, constraint (1) specifies that each party can be covered at most once. Constraint (2) refers to the capacity constraint for each tee-time. The constraints (3) and (4) specify that a reservation for a requested tee-time must occur within a specified time window. Constraints (5) enforce the model to have as many reservations as possible in one tee-time. Note that constraint (5) forces variable z_j to be one if there is an assignment to tee-time j. Constraint (6) minimizes the time deviation from the costumer's requested tee-time T_i by penalizing the deviation in the objective function. In our experiments, we considered 2 hours as the time window for a reservation, which is up to 1 hour before or 1 hour after the requested time. For example, if a customer asks for an 0848 hours tee-time, GRO allows the assignment of this request to occur at a tee-time between 0748 hours to 0948 hours. Meanwhile constraint (6) guarantees a minimum time deviation from requested 0848 hours tee-time.

The objective function in GRO maximizes the total revenue from the assignment of parties to a tee-time, and minimizes the number of tee-times that are not at full capacity. Moreover, it minimizes the time deviation from the costumer's requested tee-time. The minimization is a secondary priority for this objective function of GRO. Therefore, the value of parameter c_j, p_i^+ and p_i^- should be very small compared to r_{ij}. This cost can be an arbitrarily small value associated with variable z_j, y_i^+ and y_i^-. In our test, the value of c_j is empirically set at \$5. Similarly, p_i^+ and p_i^- are set to \$3.

Three rate categories exist for the tee-times, depending on the time of the day, for example, early morning tee-times are more expensive. The prices in each rate category include discount and regular prices, which are used to differentiate the demand with different profit potentials. Therefore, the 2 hours time window allows the GRO to move the discount category to the tee-times with a lower rate or a lower demand.

SOLUTION APPROACH

The GRO model is a linear model. This model can be directly solved by the B&B algorithm. However, our empirical results show that the direct application of B&B to this problem takes about 10 hours for each run – due to the large size of the problem – using a built-in algorithm of the B&B in the SAS-OR software. To reduce the computational time, we propose a heuristic to find an initial feasible solution to the GRO. The rationale is that the computational efficiency of the B&B algorithms can be greatly improved by having a quality initial solution (Geoffrion and Marsten, 1972).

In the following algorithm, we introduce the proposed heuristic in the form of a pseudo code. Its solution is used in the B&B method as a lower bound called Algorithm 1.

Algorithm 1: Golf Tee-Time Assignment

Initialization

Sort reservations in a non-decreasing order of arrival for tee-time and a decreasing order of party size;
Set all $x_{ij}=0$. ($x_{ij}=1$ if booking i is assigned to tee time j; otherwise the value $=0$).

Iterations

Step1	Find a feasible assignment, x_{ij}, with the maximum revenue; if a feasible assignment is not identified, go to Step 2; and then go to Step 3.
Step 2	Set $x_{ij}=1$ and go back to Step 1.
Step 3	Stop when an integer-feasible solution of the assignment is found.
Step 4	Enter the solution as a low bounder to an MIP solver to obtain an optimal solution for the problem.

The resulting solution obtained from Algorithm 1 is used as a lower bound to GRO during the B&B process. This lower bound reduces the CPU time from hours to a few minutes.

COMPUTATIONAL RESULTS

For the computational experiments, we implemented the GRO on SAS Enterprise Guide 4.0 software. We conducted the experiments using forecasted demand for a typical golf course with 9 min tee-time intervals, which allows for approximately 60 different tee-times between 0730 hours and 1500 hours. The one-day problem size for this particular golf course contains 46 500 variables and 1081 constraints. Our empirical results show that the direct application of B&B to this problem takes about 10 hours for each run (due to the large problem size) using a built-in algorithm of B&B in the SAS-OR software. For the 60-day forecast, we performed the assignments daily for 60 days in the advance base. Therefore, the 10-hour optimization is impractical for the industry.

Later, we used the heuristic explained in Algorithm 1 to find the initial integer-feasible solution to the GRO. This initial solution improved the performance of the algorithm and cut the CPU time to a few minutes. Table 1 shows that for a particular set of data, the heuristic solution assigned 69 reservations to the tee-times and generated a $15 857 revenue. By using this lower bound in the GRO, Table 2 presents the optimal solution with 88 assignment and $17 267 of revenue.

The demand data of different-sized parties are established from the simulated historical data. The demand is forecast for all incremental time intervals (tee-time intervals) of 9 min, according to the party size as shown in Table 3. The first row of Table 3 shows that the demand forecast for a party of size 1 at 0736 hours is 1. Similarly, the forecast for a party of size 2 at 0736 hours is 2.

Table 1: Heuristic solution

Total revenue	*Assigned demand*
$15 857	69

Table 2: Optimal solution

Total revenue	*Assigned demand*
$17 267	88

Table 3: Forecasted demands

Index no	*Party size*	*Forecasted demand*	*Capacity*	*Tee-time*
1	1	1	4	7:36
2	2	2	4	7:36
3	1	1	4	7:45
4	2	2	4	7:45
5	1	1	4	7:54
6	2	2	4	7:54
7	1	1	4	8:03
8	2	2	4	8:03
9	1	1	4	8:12
10	2	2	4	8:12
11	1	1	4	8:21
12	2	2	4	8:21
13	1	1	4	8:30
14	2	2	4	8:30
15	2	3	4	8:39
16	2	3	4	8:48
…	…	…	…	…
456	4	1	4	13:54

Table 4: Tee-time assignments

Tee-time	*Reservation 1*	*Reservation 2*	*Reservation 3*	*Reservation 4*
7:36	B001D1S2	B003D1S2	B003D1S2	—
7:45	B002D1S2	—	—	—
7:54	B004D1S2	—	—	—
8:03	B017D1S2	B017D1S2	B017D1S2	—
8:12	B008D1S2	—	—	—
8:21	—	—	—	—
8:30	B015D1S2	—	—	—
8:39	B006D1S2	B006D12S2	—	—
8:48	B010D1S2	B010D1S2	—	—
8:57	B021D1S2	B021D1S2	—	—
…	…	…	…	…
13:27	B085D1S4	B096D1S4	—	—
13:36	B099D1S4	—	—	—
13:45	B101D1S4	—	—	—
13:54	B103D1S4	—	—	—
13:27	B085D1S4	B096D1S4	—	—
13:36	B099D1S4	—	—	—
13:45	B101D1S4	—	—	—
13:54	B103D1S4	—	—	—

Table 4 shows the assigned reservations for the tee-times. The capacity for each tee-time is a maximum of four reservations. Therefore, for each tee-time, we are able to take up to four reservations as is shown in Table 4. For example, this means that at 0736 hours, three reservations are assigned – one with ID B001D1S2 and the other two with reservation ID B003D1S2.

CONCLUSION

As a research project, we studied a special RM problem in the golf course industry, as compared to the RM problems in the airline and hotel industries. A unique feature of this golf reservation problem is that the resources are provided in blocks, for example, a golf tee-time with a capacity of maximum four. This resource block can accommodate up to four reservations (if all the reservations are of size 1). These unique features distinguish this problem from those in the hotel and airlines industries. We propose an mixed integer programming (MIP) model to solve this problem.

With respect to the methodologies adopted in this study, the linear models in the GRO can be implemented directly and solve the problem with the B&B algorithm. To overcome the complexity of the algorithm and to solve the problem more efficiently, we propose a heuristic algorithm to find a quality-feasible solution that can serve as a lower bound in the B&B algorithm. This heuristic solution substantially reduces the CPU time for solving the problem.

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