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# A Mathematical Model to Monitor Late Arrivals at 

 Work by Junior StaffS. A. Oke \& T. M. Ezenachukwu

## ABSTRACT

Staff organisational linkages research (i.e., absenteeism, work accidents, turnover) is receiving increasing attention in the growing business competitiveness of today that requires effective utilisation of human resources. Organisations seek to hire the 'best brains', train them, and bring out the best in them so as to optimise the company's potential. Thus, organisations strive to control absenteeism in order to maximise human resources utilisation. This paper is a refreshing approach at addressing matters that relate to staff organisational linkages. The paper presents a mathematical model on the absenteeism research. The model is developed and applied in a process industry that engages in the production and sales of beer and soft drinks. The model quantifies the extra amount payable to a junior staff on an hourly basis. J unior staff who arrive early at work for productive activities would be paid an equivalent value of money for every extra hour spent using a scheme that is based on the principle of polynomial interpolation and numerical analysis which calculates the amount earned based on time. The formulation is tested to ensure accuracy of results generated during usage. This approach is fair to all staff involved since a linear model would not be favourable to staff residing far away from the company's work place location. The implications of this paper to human resources practices in contemporary organisations are many sided. However, a strong point is that the consciousness of being monitored by management has the potential to bring about the 'best' in staff, and it is also likely there would be a quest for prompt attendance and regularity at work. Possibly, there would be a strong drive not to enter into the black book' of the organisation. Consequently, the development, and application of this performance driven reform could be greatly enhanced in an information technology environment

## that displays performance contentions are discussed.

## I NTRODUCTI ON

Absenteeism is one of the classical topics in the research agenda of staff organisational linkages (Guinchard 1998, Dierendonck, Le Blanc \& Breukelen 2002). Since the early years of staff organisational problem solving, a great deal of research activities has been conducted. Fruitful results have been produced, many of which have been widely adopted in both manufacturing and service systems. In recent years the research scope of absenteeism has been greatly expanded due to the rapidly escalating competitive business arena, and the increasing number of employees who are employed in useful work in organisations (Price 1995, Guinchard 1998, Sanders 2004). A number of notable studies include the work by Sanders (2004) who focused on the relationship between informal relationships, work ethics and (short time) absenteeism. The results of that study showed a negative relationship between cohesiveness within a team and short term absenteeism of employees, as well as an interaction effect.

There have been a number of salient contributions to the stream of work related linkages. For instance, Guinchard (1998) presents a set of propositions concerning aspects of the relationship between psychoanalysis and firms. His paper explains the possible role of phantasy in the context of the firm and employs this approach to examine the issue of employee absenteeism. In a different line of enquiry, Dierendonck, et al. (2002) tested a model that links supervisory behaviour and reciprocity in the supervisor/subordinate relationship to objectively register subordinate absenteeism. Supervisory conflict management behaviour was not significantly related to the subordinates' feelings of reciprocity, but only indirectly leader member exchange through (LMX) behaviour. Earlier, Price (1995) proposed a role for demographic variables in the study of absenteeism and turnover. That paper advances the argument that there are two inappropriate uses of these variables in current research, as variables in causal models and as measures. These contributions demonstrate a variety of approaches.
A great deal of the literature has been concerned with working hours and its effects. For example, Worrall and Cooper (1999) examined the patterns of actual working hours. Their analysis revealed a strong relationship between actual hours worked and an increasingly negative impact on all the factors tested. Olorunsola and Ibegbulam (2003) report on a convenient working schedule for librarians. Their study advocates the implementation of a new working arrangement for librarians in order to tackle this problematic issue of working schedule. Despite the usefulness of these two studies, no attempt has been made to discuss the issue of working hours as it relates to late arrival at work. Furthermore, Hetty van Emmerik and Sanders (2005) recently examined the relationship between preferred and actual number of hours, and commitment. In their analysis it was argued that women and part time working employees attach more importance to their working hours. And in a geographically dispersed location, Fujimoto (2006) explored how J apanese workers allocate their time to work and private life in different stages of life. The results of that study indicate that a majority of J apanese workers have a strong preference to work constantly without major career disruptions between the ages of 20 to 60 years. However, this reduces drastically the time spent in paid work after job retirement at age 60. A limitation of that study as well as the Hetty van Emmerik and Sanders (2005) investigation is that the concept of late arrival at work was not addressed.

Attempts have been made to make working hours flexible. In the case of the white collar workers and in the office environment flexitime arrangements have been offered to employees so that regular services can be available to customers or client without interruptions. This situation is difficult to implement in manufacturing environments due to the nature of work, which would warrant organising work shifts. For instance, during the production activities in manufacturing, the storekeeper, maintenance team and security personnel are key staff that need to be present to support production activities. The storekeeper releases items from store(s) for production, the
maintenance team would be available for healthy upkeeping of machinery and equipment, and the security personnel monitors the movement of people and other resources. Nevertheless, this paper focuses on the manufacturing context where a particular number of workers may be present at a certain time period of the day, and hence, the punctuality and full attendance is very important. Indeed, few absences are a strong indicator for managing tardiness. Other studies have shown how this practice of flexitime can be effective (Worrall \& Cooper 1999, Olorunsola \&Ibegbulam 2003). In addition, the average wages paid to the employees are among the industrial best rates in that particular city. In addition, the firm has a common transportation system, like buses to pick up employees along the route and to make sure they arrive at work on time. However, the limitation of this transportation service is that it did not cover all major routes that lead to the firm.
The few routes covered are determined by the concentration of employees living along those routes. However, this transportation system has its own problems, as evidenced from the past experiences of other firms. For example, during a particular crisis situation the buses from a major multinational oil production company were to be set ablaze, but fortunately, the law enforcement agents intervened. The excuse from the angry mob was that the firm makes huge revenues from the society without 'ploughing back' some of the profits to the community in order to improve the standard of community living. Thus, such a transportation service needs to be carefully managed. For instance, it could be contracted out to other firms for effective management. In addition, the use of a company logo, which may attract undue attention of an angry mob during crisis period, may be avoided. Other measures that the firm has implemented include developing strong teamwork among employees as this may increase job satisfaction, and hence, the enthusiasm of employees to come to work and meet their team members. There are several other methods that the management has adopted to make the management issue of absenteeism manageable, which include developing career paths and/ or providing relevant training for their career growth, that may engender some work aspirations and disciplines among the workers.

Employees may be properly managed and monitored on the issues of absenteeism and tardiness if the profit maximisation objective of the organisation is to be achieved. Monitoring is necessary due to at least four reasons: (1) To ensure that junior workers' input to a company's productive output is not undermined, (2) Managing late arrivals by way of extra pay for arriving earlier than normal time is also a form of motivation for workers, (3) Monitoring late arrival is a means of knowing the amount of man hours wasted as a result of late arrival and this can be translated into loss in output for the purpose of a company's financial projections, and (4) Managing late arrival by way of extra pay instead of threats to sack people helps to ensure that junior staff work in harmony with their superiors and this creates a corporate accord between workers and superiors, which is a necessary ingredient for productivity. Consequently, if the modern organisation is to be driven to a position of superiority and acquire world class bench marks employees must be well motivated to achieve regular work attendance and maintain good product quality, to build low cost products, enhance company competitiveness, and be effectively utilised. Granted that the workforce is of the right quality and number, there is a compelling need to properly control staff so that delivery datelines are always met, and the goodwill of the organisation is preserved.

In the current decade, new technologies are emerging every day, particularly in the process industry that engages in the production and sales of beer and soft drinks. These new technologies also bring new challenges in their management using the operational level manpower available to companies. Despite a great deal of exciting research being conducted to identify and address workplace challenges the area of monitoring late arrivals at work by junior workers remains unresolved (Worrall \&Cooper 1999). Thus, the focus on junior staff is justified.

Late arrivals at work may exist due to various reasons. For instance, the organisational culture may require refining, the system design and its implementation may require reforming, and managers may seek to resolve issues in a relatively reactive way by controlling or monitoring mechanisms. However, the redesign of the system, changing the culture and motivating employees are complementary and effective ways to control, monitor, and manage late arrival at work. Junior staff are likely to take advantage of weak management mechanisms which allow employees to
arrive at work late without a penalty. While such tardiness may be displayed by all staff, lateness to work can be a phenomenon particularly of junior staff who may have not learned the 'desirable' culture of arriving at work on time. Thus, there is need to improve the supervision of work starting time as curbing of late arrivals practices at work is an organisational linkage problem.
This paper centres on an innovative approach to managing late arrivals at work in a process industry that engages in the production of beer and soft drinks. The current problem has been challenging to the company management since much frustrations are usually encountered with getting the junior staff to produce orders after the marketing department has obtained the customer's orders for production. Thus, much income has been lost by the company to this unsatisfactory behaviour of workers with respect to attendance. The company management also noticed that the goodwill of the company is declining. Another challenge posed to management by the late arrival of junior workers is the unpredictability and instability of the company's budgeting activities. On many occasions it is embarrassing for the head of production who has to convince the accountant on the manpower budget for the month since workers exhibit irregular attendance at work. The focus on junior staff is warranted as they occupy a large proportion of the plant workforce.

The paper has five sections. In the first section is presented a late arrivals monitoring framework. The second section details the methodology that was used to evaluate the manuscript theme. A results sector, the third section of the paper, is followed by a discussion (i.e., the fourth section), and the last section is a conclusion. The section for the late arrivals monitoring framework presents a detailed breakdown of the problem of tardiness as it relates to a process industry. The methodology section presents about the subjects (respondents), the study sites, and the procedure for carrying out the study. The results section presents the results of the analysis obtained, and the discussion section outlines the practical and operational elements of the model. The last section of this paper gives concluding remarks in terms of the applicability of the model presented and its benefits. Some possible future extensions of the model are also suggested.

## The Late Arrivals Monitoring Framework

This section provides the theoretical underpinning of the theme of the paper. The problem of absenteeism in the study site is so serious that management from the international head office of the company, in Geneva, had requested the site manager to explain why production figures had decreased during the previous three months without any major plant overhaul programme being implemented. As a result, the site manager summoned all the top management staff to an emergency meeting where ideas concerning how to solve the problem of absenteeism were discussed. A first issue raised was the large number of junior staff who were engaged in unauthorised full time degree programmes in higher institutions. These staff were absent for several days on the 'platform' of having casual leave, which the staff then converted (illegally) to examination leave. Consequently, at this management meeting it was resolved that staff were not permitted to enrol for any university degree programme without authorisation from the management.

A second issue that was raised at the crisis meeting was the monotonous nature of work, which makes the job uninteresting to staff. After considerable discussion, it was agreed that any staff were not to spend more than a set number of months on a particular task, and that job redesign, in the form of job rotation was to be installed. The period of job rotation was to be determined by the personnel manager by studying jobs, their contents, skill requirements, and the available range of work settings.
A third issue raised was the causes of absenteeism. One salient feature that was identified is that the workers' welfare package is poor. On a number of occasions when staff members or their family members fell sick, the need to seek external healthcare assistance was a distraction to work. To resolve the absenteeism/late arrival problem, that was related to health issues, the management agreed to establish a scheme to reward better attendance. One key plank of the strategy was to give
financial and non financial rewards such as a 25 per cent bonus on salary of the concerned staff who promptly attend work, and a second plank was to have better attendances named on the 'Roll of Honour', which is a list created to appreciate attendance at work. Consequently, to encourage junior staff to attend the work place a financial reward system has been implemented for arriving early. Attending their work system before the commencement time of $8.00 \mathrm{a} . \mathrm{m}$. and being engaged in productive work is financially rewarded (Allan, Brosnan \&Walsh 1998).

The challenge is to develop a system that monitors the arrival time of every junior staff member. A computer system will monitor attendance and effectively calculate the extra amount payable to staff, monthly by the number of hours added to normal working hours by arriving early. There are various electronic displays mounted at different strategic positions within the work site to remind members of staff of issues relating to meeting hourly targets, and ultimately assigned targets of the day. A very good example of such targets is that given to the operator in charge of the unpacker machine. The target is to ensure that the machine unpacks 30 pallets of crates per hour. One pallet has 70 crates on it and each crate of Dyna Beer has 12 bottles in it, amounting to 840 (i.e., $70 \times 12$ ) bottles. This target of 30 pallets per hour would have been determined from the company's targeted profit for the year. Therefore, from the profit (set as a goal) every staff member is assigned targets to meet in order to ensure that at the end of the fiscal year the declared profit may be equal to or greater than the set target.
This computer system monitors the arrival times of every junior staff member on every working day and then calculates the total extra hours to be paid to each staff for arriving early. The amount to be paid is added to the staff member's monthly salary. This paper presents 'a solution' to the absenteeism problem from the perspective of calculating and modelling the amount of time and payment. The initiative uses polynomial interpolation to generate a model of the problem, and if implemented, may resolve the issue of late attendance. There are four features to the system; (i) Keeping track of arrival, (ii) Calculating of total number of extra hours, (iii) Calculating of the amount payable to staff with respect to payment, and (iv) Modelling of the equation relating shuttling time and amount to be paid per hour.
This paper is concerned with the fourth feature (i.e., the modelling of an equation relating the ' travel' time of staff to the amount the staff is to be paid).

Let $y=$ the amount (in naira) to be paid for every extra hour worked $\mathrm{t}=$ time it takes a staff to shuttle from his/ her residence to the office

A relationship was derived between $y$ and $t$ such that $y$ would be a function of $t$ (i.e., $y=f(t)$ ), using a numerical analysis and a polynomial interpolation method. In using polynomial interpolation, a number of data points are required to serve as starting points. Table 1 shows some data points chosen for $y$ and $t$ based on a fair consideration of average hourly salary and worked time.

$$
\begin{gathered}
\text { Table } 1 \\
\begin{array}{c}
\text { Data Point for y and } \mathbf{t} \\
\text { Based on Average } \\
\text { Hourly Salary and } \\
\text { Time }
\end{array} \\
\hline \text { y (A/ hr) t (mins) } \\
\hline 0 \mathrm{y}\left(\mathrm{t}_{0}\right)=90 \quad \mathrm{t}_{0}=20 \\
1 \mathrm{y}\left(\mathrm{t}_{1}\right)=190 \mathrm{t}_{1}=50 \\
2 \mathrm{y}\left(\mathrm{t}_{2}\right)=250 \mathrm{t}_{2}=130
\end{gathered}
$$

Table 1 shows that if it takes a staff member 20 minutes to shuttle from residence to the office, then the employee may receive $\# 90$ for every hour he arrives earlier for work. The equation relatingy and t is useful in the determination of this monetary value. However, for other staff that are known to shuttle for 30 minutes, 80 minutes and 150 minutes, the value of y can be calculated. Employing
the theory of numerical methods and polynomial interpolation the relationship between $y$ and $t$ can be expressed as is shown in equation 1.

$$
\begin{equation*}
y_{k}\left(\mathrm{t}_{\mathrm{i}}\right)=\sum_{\mathrm{k}=0}^{\mathrm{n}} \mathrm{y}\left(\mathrm{t}_{\mathrm{k}}\right) \mathrm{L}_{\mathrm{k}}(\mathrm{t}) ; j=0,1,2, \ldots, \mathrm{n} \tag{1}
\end{equation*}
$$

This means that the amount paid for every hour worked is an aggregate of elements of time worked at different periods, where $L_{k}(t)=q_{k}(t) \div q_{k}\left(t_{k}\right)$, which means the ratio of $q_{k}(t)$ to $q_{k}\left(t_{k}\right)$.

$$
\begin{equation*}
q_{k}(t)=\prod_{\substack{j=0 \\ j=k}}^{n}\left(t-t_{j}\right) \tag{2}
\end{equation*}
$$

Equation 2 is the product of the difference between the time it takes a staff to shuttle from residence to the office, $t$ and the target period, $\mathrm{t}_{\mathrm{j}}$.

$$
q_{k}\left(t_{k}\right)=\prod_{\substack{j=0 \\ j=k}}^{n}\left(t_{k}-t_{j}\right) ; k=0,1,2, \ldots, n(3)
$$

Equation 3 is the product of the difference between the arrival period and the target period. Note that $\Pi$ denotes product not summation.
For this case, three data points are obtained such that the concern is limited to $\mathrm{k}=0,1,2$ and $\mathrm{j}=$ 0,1,2.

Now, for $k=0, q_{0}(t)=t^{2}-180 t+6500$ and $q_{k}\left(t_{k}\right)=q_{0}\left(t_{0}\right)=3300$.
For $\mathrm{k}=1, \mathrm{q}_{\mathrm{k}}(\mathrm{t})=\mathrm{q}_{1}(\mathrm{t})=\mathrm{t}^{2}-150 \mathrm{t}+2600$ and $\mathrm{q}_{\mathrm{k}}\left(\mathrm{t}_{\mathrm{k}}\right)=\mathrm{q}_{1}\left(\mathrm{t}_{1}\right)=-2400$.
Fork $=2, q_{k}(t)=q_{2}(t)=t^{2}-70 t+1000$ and $q_{k}\left(t_{k}\right)=q_{2}\left(t_{2}\right)=8800$.
Nowrecall that $L_{k}(\mathrm{t})=\mathrm{q}_{\mathrm{k}}(\mathrm{t}) \div \mathrm{q}_{\mathrm{k}}\left(\mathrm{t}_{\mathrm{k}}\right)$.
Therefore, $\mathrm{L}_{0}(\mathrm{t})=\mathrm{q}_{0}(\mathrm{t}) \div \mathrm{q}_{0}\left(\mathrm{t}_{0}\right)=\mathrm{t}^{2}-180 \mathrm{t}+6500 \div 3300$,
and $\mathrm{L}_{1}(\mathrm{t})=\mathrm{q}_{1}(\mathrm{t}) \div \mathrm{q}_{1}\left(\mathrm{t}_{1}\right)=\mathrm{t}^{2}-150 \mathrm{t}+2600 \div-2400$.
Also, $\mathrm{L}_{2}(\mathrm{t})=\mathrm{q}_{2}(\mathrm{t}) \div \mathrm{q}_{2}\left(\mathrm{t}_{2}\right)=\mathrm{t}^{2}-70 \mathrm{t}+1000 \div 8800$.
But our objective is $\mathrm{y}(\mathrm{t}) \stackrel{2}{=} \mathrm{k}=0$ to n of $\mathrm{y}\left(\mathrm{t}_{\mathrm{k}}\right) \mathrm{L}_{\mathrm{k}}(\mathrm{t})$, which nowbecomes
$\mathrm{y}(\mathrm{t})=\mathrm{y}\left(\mathrm{t}_{0}\right) \mathrm{L}_{0}(\mathrm{t})+\mathrm{y}\left(\mathrm{t}_{1}\right) \mathrm{L}_{1}(\mathrm{t})+\mathrm{y}\left(\mathrm{t}_{2}\right) \mathrm{L}_{2}(\mathrm{t})$. Substituting for $\mathrm{L}_{1}(\mathrm{t}), \mathrm{L}_{2}(\mathrm{t})$ and $\mathrm{L}_{0}(\mathrm{t})$, it nowimplies that
$y(t)=90\left[t^{2}-180 t+6500 \div 3300\right]+190\left[t^{2}-150 t+2600 \div-2400\right]+250\left[t^{2}-70 t+1000 \div 8800\right]$
and

$$
\begin{equation*}
y(t)=\frac{-62 t^{2}+13140 t-400}{2640} \tag{4}
\end{equation*}
$$

The equation $\mathrm{y}(\mathrm{t})$, shown as equation 4, is the mathematical model of the problem represented. Now the value of $y$ can be obtained for any shuttling time $t$. Therefore, the amount of money payable to a staff for arriving earlier than normal start of work can be determined if the shuttling time t, is known. The next section of the paper deals with the testing of the model for its validity. It also shows how it can be applied to address the problem of work attendance. Installing an automated system that would measure the effectiveness of this management mechanism (model) is a worthwhile venture that is pursued in the Methodology.

## Methodology

## Subjects

The subjects for the research work were carefully chosen. Their relevance is embedded in the feature that these employees have the responsibility of either producing units of products or rendering a service, which would adversely affect the whole plant when they are absent. And, the focus is on the junior staff in the plant. This category of staff is distributed around the plant for
several activities, which include production, maintenance, accounting, security, personnel, sales, stores and utilities. The coverage entails all the junior staff operating in these functions (which are not only manufacturing, but also support or service activities within the company). Nevertheless, if the machines are poorly maintained or not performing optimally the production targets are unlikely to be attained. Consequently, absence from duty would drastically affect production, quality of service rendered, or put the organisation at a risk of losing money or goodwill. For example, a target to deliver 250,000 cartons of product in two weeks will not be met if the operator on the bottle washer is conspicuously absent for two weeks when there are no other skilled staff to operate the machine. Also, if the quality control staff (who check defective products) is absent without notice, the company risks passing a defective product into the market, which may result into a legal liability that may adversely affect company sales or even close down the plant temporarily. Such absenteeism may embarrass the company, and subsequently, lead to a sanction by the appropriate government authorities. The age range of these people is usually between 18 and 60 years. The qualifications required of this personnel ranges from Senior Secondary School Certificate, Professional Vocational Certificates and a maximum of Ordinary National Diploma.

## Site

The case considered here relates to a hypothetical process industry, christened Dynamics Limited, that engages in the brewing process and soft drinks production. Dynamics was initially formed to import Dyna into the country from Geneva. The popularity of the Dyna brand led to the establishment of a Dynamics brewery in Lagos in 1973. Notably, this was the first Dynamics brewery outside Geneva, and only the fourth in the world. Dynamics has recorded tremendous growth in production over the years. Today, it has three breweries in the country, and apart from its flagship Dyna product the company also produces soft drinks. Dynamics is a capitalised company in the Nigerian Stock Exchange. Dynamics has 3.5 per cent of the total market capitalisation of the stock exchange, and is located in the commercial nerve centre of Nigeria, Lagos, where it produces an average of 10 million units of products annually. The organisation is a multinational company managed by well tested, goal setting, results getting, and target oriented top management executives, who ensure that 'all hands are on deck' to get the various jobs and tasks done.
The company had a major extension in 1980 with the acquisition of 'large size' hectares of land. The company's policy is set to achieve a number of goals in quality, flexibility, delivery, and cost. The company sets a taste score of 3.0 or below for quality attainment. For flexibility, an operational performance index of 52 per cent is set as a minimum. The delivery goal of the company is set at on time delivery for 70 per cent of all orders made. For the cost objective, a 200 hour per full time employee per year work hour is set.
Dynamics Limited sets targets on the total number of units of a particular product that may be produced daily. Meeting these daily targets of say X crates of beer per day depends on the efficiency of the junior staff directly involved with the machines (the technicians, the operators, packaging staff). In many manufacturing organisations, junior staff population is an important component of the organisation's staff list, but unfortunately, they can be difficult to manage. In fact, their attendance at work is motivated by many factors, which when absent in a system, can lead to a severe dysfunctional organisational outcomes.

There is a huge demand for the company's products nationwide and even outside the nation's borders. Therefore, apart from the company's managerial tasks, of immense importance is daily output in terms of market units. This is important as the company strives not only to meet increasing daily demand, but to also serve anticipated high demand of new markets. In order to meet these demands, the company has over the years established six brewery plants at strategic cities in the country. To aid the company in meeting demand from neighbouring countries, and in a quest to discover new/ emerging markets, the company has recently established another brewery plant in Sierra Leone. The issue of discourse in this paper, therefore, is aligned with meeting
demand by producing desirable levels of goods and targeting those staff directly involved in the production processes.

## Procedure

This study has six elements. First, the organisation must have an effective record system, which contains names and personal details of all employees. Such a system may be maintained either manually or with automatic machines. Second, inform all the employees about the programme(i.e., staff enlightenment), stating the benefits of such a programme to the organisation and its financial benefit to individuals. Third, implement the system by taking note of arrival and departure periods for all days. Fourth, compute the financial consideration due to employees at the end of month. Fifth, give employees and company management feedback on their performance. Last, repeat this cycle until the results improve.

## Results

The last section involved the derivation of the much needed symbolic (mathematical) model. This model is shown as equation 10 .

```
x y(t)=(-62t squared + 13140t - 400)
    divided by 2640 is Equation }1
```

Table 2 shows the details of the calculations that involve time utilisation, which are expressed in minutes, and the average hourly salary, which is expressed in $\# /$ hour. The content of Table 2 indicates data points from time value of 20 to 50 minutes. For instance, at time 20 minutes, the reward is $\$ 90$. Also, for time 21 minutes, 22 minutes, and 23 minutes, the remuneration is $\# 94$, \#98 and $\# 101.9$; respectively.

Table 2

| Average Hourly Salary |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Against Time Utilised With I ntermittent <br> Data Points |  |  |  |
| Time (t) | Salary (y) | Time (t) | Salary (y) | Time (t) | Salary (y) |
| 20 | 90.0 | 30 | 128.0 | 40 | 161.4 |
| 21 | 94.0 | 31 | 131.6 | 41 | 164.4 |
| 22 | 98.0 | 32 | 135.1 | 42 | 167.5 |
| 23 | 101.9 | 33 | 138.5 | 43 | 170.4 |
| 24 | 105.8 | 34 | 141.9 | 44 | 173.4 |
| 25 | 109.6 | 35 | 145.3 | 45 | 176.3 |
| 26 | 113.4 | 36 | 148.6 | 46 | 179.1 |
| 27 | 117.1 | 37 | 151.6 | 47 | 181.9 |
| 28 | 120.8 | 38 | 155.1 | 48 | 184.6 |
| 29 | 124.8 | 39 | 158.2 | 49 | 187.6 |
|  |  |  |  | 50 | 190.0 |

Notes:
a. Time = time utilised, and Salary = average hourly salary.
b. $t$ is in minutes, and $y$ is in $A$.

Since $y(t)=\left(-62 t^{2}+13140 t-400\right) \div 2640$ (equation (10)), then
$y(40)=\left(-62(40)^{2}+13140(40)-400\right) \div 2640=(-99200+525600-400) \div 2640=426000 \div$ $2640=161: 36$
This result of 161.36 , which is shown in Table 3, demonstrates if a staff shuttles for 40 minutes before getting to work, then the extra amount to be paid for arriving one hour earlier than the normal start of work is $\# 161: 36 / \mathrm{hr}$. Note that this value falls between $\# 90 / \mathrm{hr}$ and $\AA 190 / \mathrm{hr}$, which

Table 3
Values of y ( $\# / \mathbf{h r}$ ) against t (mins)

| $\mathbf{y}(\mathrm{A} / \mathbf{h r})$ | $\mathbf{t}(\mathbf{m i n s})$ |
| :---: | :---: |
| $0 \mathrm{y}\left(\mathrm{t}_{0}\right)=90$ | $\mathrm{t}_{0}=20$ |
| $\mathrm{y}(\mathrm{t})=161: 36 \mathrm{t}=40$ |  |
| $1 \mathrm{y}\left(\mathrm{t}_{1}\right)=190$ | $\mathrm{t}_{1}=50$ |
| $2 \mathrm{y}\left(\mathrm{t}_{2}\right)=250$ | $\mathrm{t}_{2}=130$ |

Note: $y(t)$ is shown as the application of equation 10 for $t=40$ minutes.
Confirming the validity of this symbolic model, the value that has already been known will betaken as the input, say for $t=130$ minutes. From Table 3, y(130) is $250 / \mathrm{hr}$. The model would be confirmed valid if $y(130)$ gives $\# 250 / \mathrm{hr}$ from the equation as is now tested.
$y(t)=\left(-62 t^{2}+13140 t-400\right) \div 2640$, noting that $\mathrm{y}(130)=\left(-62(130)^{2}+13140(130)-400\right) \div 2640=250$.
Since this verifies to what is in Table 3 the validity of the model is confirmed. Furthermore, the model equation $\mathrm{y}(\mathrm{t})=\left(-62 \mathrm{t}^{2}+13140 \mathrm{t}-400\right) \div 2640$ (i.e., equation 10 ) as observed is quadratic and as is common to all quadratic curves, if a plot of equation 10 is given, it may be made up of an increasing portion and a decreasing portion. A graph of $y(t)$ against $t$ for the model is shown as Figure 1.

Figure 1
A Graphical Plot of $\mathbf{y}(\mathbf{t})$ Against $\mathbf{t}$
x A Graphical Plot of $\mathrm{y}(\mathrm{t})$ Against t

Figure 1 shows the range for the increasing portion is $0.03<=\mathrm{t}<=106$ and that for the decreasing portion is $106<=\mathrm{t}<=21.905$ for $\mathrm{y}>=0$. For values in the range $0.03<=\mathrm{t}<=106$ there is an increase in the value of $y(t)$ for every increase in $t$. This makes sense as an employee who takes more time to arrive to work may be paid more for every extra hour he/ she adds to work than another employee that takes lesser time. The rule of getting more pay for more shuttling time $t$ is, however, violated within the range from $t>106$ mins. Worse still are cases from $t>=211.905$ as $y$ ( t ) becomes less than zero. In very clear terms for example, applying the model for $\mathrm{t}=300$ minutes gives $\mathrm{y}(\mathrm{t})=-620.61$ which means that a long distance employee that takes 300mins (5hrs) to get to work owes the company $\# 620.61$ for every hour added to work for arriving earlier to work (this does not make sense). The model equation, therefore, fails for $t>=106 \mathrm{mins}$. In order to correct this anomaly, the model curve is mirrored about the liney $=263.56$ in order to ensure that increase in extra pay per hour $y(t)$ is maintained as the shuttling time $t$, increases. Figure 2 shows the plot
after the range $\mathrm{t}>106$ is mirrored about the line $\mathrm{y}=263.56$.
Figure 2
The Plot After the Range $\mathbf{t} \boldsymbol{> 1 0 6}$ is Mirrored About the Line $\mathbf{y}=263.56$


The model is now made up of two curves each taking care of two different ranges. The new model can now be redefined as follows.

$$
y(t)=\left\{\begin{array}{lll}
\mathrm{f}_{1}(\mathrm{t}) & ; & 0.03 \leq \mathrm{t} \leq 106 \\
\mathrm{f}_{2}(\mathrm{t}) & ; & \mathrm{t} \geq 106
\end{array}\right.
$$

And the equation 10 is $f_{1}(t)$ as $\left(-62 t^{2}+13140 t-400\right) \div 2640$.
The next step is to derive an equation for the range $t>=106$ which is defined as $f_{2}(t)$. For $f_{1}(t)$, the curve ' $n$ ' shaped as the coefficient of $t^{2}$ is negative (i.e., -62). Using $f_{1}(t)$ and making the coefficient of $t^{2}$ to be positive the expression whose curve would be ' $u$ ' shaped. Applying this outcome and assuming the new expression to be

$$
f_{3}(t)=\frac{62 t^{2}-13140 t+400}{2640}
$$

establishes Figure 3 which is a plot of $f_{1}(t)$ and the new $f_{3}(t)$ from $f_{1}(t)$.
Figure 3


Figure 3 shows that $f_{3}(t)$ is actually a mirror of $f_{1}(t)$ about the $t$ axis with points $A$ and $B$ as turning points of $f_{1}(t)$ and $f_{3}(t)$, respectively. From the figure, it is readily obvious that moving the curvey $=f_{3}(t)$ upwards such that point $B$ now becomes coincident with point $A$, the curve $y=f_{3}(t)$ becomes $f_{2}(t)$. This is achieved by giving $y=f_{3}(t)$ as displacement of 527.12727272 upwards (i.e., by adding 527.2727272 to $\mathrm{y}=\mathrm{f}_{3}(\mathrm{t})$. Consequently, $\mathrm{f}_{2}(\mathrm{t})=\mathrm{f}_{3}(\mathrm{t})+527.13$ (i.e., $\left.f_{2}(t)=\left(62 t^{2}+13140 t+400\right) \div 2640+527.13\right)$.

Put more favourably, $\mathrm{f}_{2}(\mathrm{t})=\left(62 \mathrm{t}^{2}-13140 \mathrm{t}+1392016\right) \div 2640$, which consolidates the model for the range $\mathrm{t}>=106 \mathrm{mins}$.
The final definition for the model can thus be put forward as follows.

$$
\frac{-62 t^{2}+13140 t-400}{2640}
$$

$$
y=\frac{62 t^{2}-13140 t+1392016}{2640} ; \quad t \geq 106
$$

This equation ensures that $\mathrm{y}(\mathrm{t})$ is always increasing as shuttling time t increases. Using four testing values of $\mathrm{t}=40,80,120$ and 150 minutes gives the following results.
Fort $=40$ mins, $y(t)=\left.f_{1}(t)\right|_{t=40}=\frac{-62(40)^{2}+13140(40)-400}{2640}=A 161.36 / \mathrm{hr}$
Fort $=80$ mins, $y(t)=\left.f_{1}(t)\right|_{t=80}=\frac{-62(80)^{2}+13140(80)-400}{2640}=\$ 247.73 / \mathrm{hr}$
x $\mathrm{f} 2(\mathrm{t})$ where $\mathrm{t}=120$

For $\mathrm{t}=120 \mathrm{mins}, \mathrm{y}(\mathrm{t})=$ $\qquad$ $=\#$ 268.19/hr


These analysed values show a general increasing trend for extra hourly pay $y(t)$ as $t$ increases.

## Discussion

This study presents a framework that monitors late arrivals at work by junior staff in a process industry that engages in the production and sales of beer and soft drinks. The commercial environment of the location of the company does not permit workers to live in its immediate environment. This poses a challenge to workers of overcoming traffic congestion problems everyday before arrival at work. Thus, workers may have to leave their residential locations early to arrive at work on time. Consequently, the model, which utilises interpolation and numerical analysis techniques present a fairer evaluation of staff residing in the city, irrespective of their locations. This is a strong feature of the mathematical model utilised as against a linear model structure that may present an unfair assessment in which employees living far from the company's location are at a disadvantage, while those living nearby have an advantage in terms of getting to work earlier.
At the implementation of this model it is noted that solving the tardiness problem requires an understanding of the organisational culture, system design and implementation at the company investigated. Only then is the implementation of the late arrival monitoring model useful. The implementation team may be able to blend behavioural issues, which are qualitative (i.e., organisational culture) with quantitative issues such as measurement, and analysis of arrival statistics. At the implementation stage, a good organisational climate is first provided, with managers and supervisors adequately conversant with respect to arriving early at work. Clearly, it is difficult for junior staff to report to managers and supervisors to who may arrive late at work. Thus, the consciousness has been created in the employees that a good organisational chine in the company requires prompt attendance of staff to the workplace.
A strong technique in motivating the workers to attend work on time is the use of electronic displays. These facilities remind people about their responsibilities, the goals of the company and the possible benefits that the company would gain when workers embrace a good work culture. It is in this atmosphere, where everybody knows what is expected of him or her, and the possible reward that emanates from compliance with company's goals, that the model presented and its application is likely to flourish. The company is also careful to improve its image in the public. The poor performance records associated with the company in recent times needs to be corrected so that confidence can be restored to the management.
From Table 1 it is observed that different time considerations are given to all staff on the account that the company staff resides in various locations in the town. This may be due to economic or some other reasons. Thus, for a staff who resides at Surulere (where the company is situated), it may take just 20 minutes to shuttle between residence and office. However, for workers staying at more geographically remote places (such as Iyana Ipaja, Sango-Otta or Ikorodu), the amount of time spent in shuttling between residence and the work office may be about 80 minutes and 150 minutes; respectively. With these residential locational differences, a fair approach is, therefore, required in the computation of the amount earned by staff based on time. Thus, the principle of polynomial interpolation and numerical analysis is utilised. The approach is fair to all staff involved since a linear model would be unfavourable to staff residing far away from the work place.

## Conclusion

The theme of this paper centres on the need to monitor and control junior staff attendance in terms of late arrivals at work in a process industry. In particular, this paper discusses the use of mathematical modelling to implement a company's management policy on discouraging lateness to work. The model relates the extra amount that junior staff are paid by working some extra hours with respect to arriving earlier for work. Application of the model has potential to influencejunior staff work attendance patterns.

The content of this paper has implications for contemporary organisations. First, records on performance of individual junior staff will need to be kept over a long period in order to develop historical records for employees. Such records are useful for several organisational decisions. An immediate usefulness of such historical records is to create a reward system for high performers. In addition, these data sets can be expanded to add other information about staff such as skill, competence, discipline at work, and teamwork attributes. This information may be used in order to promote staff. The company could also use the historical information of the staff attendance register for planning customer orders in the future. Thus, a more effective decision can be made on whether or not a production order placed by the marketing department should be taken from the outset. Another implication of the model presented is that information on attendance could be used for purposeful integration of HRM policies and practices.

The residual data in the organisational records can have a variety of pragmatic issues. For product expansion purposes, or in meeting delivery datelines, management could predict the possible time it would require to meet up with a certain order delivery demand. Another practical implication of the model is that information collected from its implementation could assist organisations in deciding on what staff to disengage during downsizing events. Staff who fail to measure up to the request for timely arrivals at work may be marked for possible layoff during organisational downsizing activities. The results obtained may provide a strong point in creatingjob alertness, and thus, a quest for efficiency through delivery of services in a timely manner. The compliance of individual staff to this measurement scheme may provide a strong indicator of workers' commitment to work, and possibly reflect their ability to cope with challenges in the workplace in contemporary organisations. Obviously, an employee with a consistently low performance on attendance may imply a declining productivity performance. Such employees may have a myriad of family problems, which may prevent greater concentration work, and consequently, face the risk of accidents. Obviously, committed staff who attend work regularly need to be rewarded financially and even through promotion.
Since the probability of a junior staff arriving at work is uncertain, a number of factors that affect arrival at work could be built into the model to make it more realistic. Thus, extension of the model may incorporate fuzzy logic principles to capture possible failure statistics of transport facilities that the staff would have to board to work. Also, some non easily quantified factors such as domestic problems, which may influence workers' arrival times at work may be integrated into the model. Nevertheless, there is much promise for the development of the concept proposed in this work. Although the junior staff category is chosen for the study, future work could extend it to supervisory cadre of staff on the plant or other categories that are deemed fit.

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