On a role of predictor in the filtering stability

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Abstract
When is a nonlinear filter stable with respect to its initial condition? In spite of the recent progress, this question still lacks a complete answer in general. Currently available results indicate that stability of the filter depends on the signal ergodic properties and the observation process regularity and may fail if either of the ingredients is ignored. In this note we address the question of stability in a particular weak sense and show that the estimates of certain functions are always stable. This is verified without dealing directly with the filtering equation and turns to be inherited from certain one-step predictor estimates.

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