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Survival probabilities for branching Brownian motion with absorption

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Abstract

We study a branching Brownian motion (BBM) with absorption, in which particles move as Brownian motions with drift $-rho$, undergo dyadic branching at rate $beta>0$, and are killed on hitting the origin. In the case $rho>sqrt{2beta}$ the extinction time for this process, $zeta$, is known to be finite almost surely. The main result of this article is a large-time asymptotic formula for the survival probability $P^x(zeta>t)$ in the case $rho>sqrt{2beta}$, where P^x is the law of the BBM with absorption started from a single particle at the position $x>0$. We also introduce an additive martingale, V, for the BBM with absorption, and then ascertain the convergence properties of V. Finally, we use V in a `spine' change of measure and interpret this in terms of `conditioning the BBM to survive forever' when $rho>sqrt{2beta}$, in the sense that it is the large t-limit of the conditional probabilities $P^x(A|zeta>t+s)$, for $AinF_s$.

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