Asymptotic Distribution of Coordinates on High Dimensional Spheres

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Abstract
The coordinates x_i of a point $x = (x_1, x_2, \dots, x_n)$ chosen at random according to a uniform distribution on the $l_2(n)$-sphere of radius $n^{1/2}$ have approximately a normal distribution when n is large. The coordinates x_i of points uniformly distributed on the $l_1(n)$-sphere of radius n have approximately a double exponential distribution. In these and all the $l_p(n), 1 \le p \le \infty$, convergence of the distribution of coordinates as the dimension n increases is at the rate $n^{1/2}$ and is described precisely in terms of weak convergence of a normalized empirical process to a limiting Gaussian process, the sum of a Brownian bridge and a simple normal process.

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