Diagnostic Tests for Non-causal Time Series with Infinite Variance

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Abstract

We study goodness-of-fit testing for non-causal autoregressive time series with non-Gaussian stable noise. To model time series exhibiting sharp spikes or occasional bursts of outlying observations, the exponent of the non-Gaussian stable variables is assumed to be less than two. Under such conditions, the innovation variables have no finite second moment. We proved that the sample autocorrelation functions of the trimmed residuals are asymptotically normal. Nonparametric tests are also investigated. The rank correlations of the residuals or the squared residuals are shown to be asymptotically normal. Thus, an assortment of portmanteau statistics are available for model assessment.

Keywords: Non-causal AR Process; Infinite Variance; Goodness-of-fit; Portmanteau Test; *alpha*-stable distribution.

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1 Introduction

Infinite variance autoregressive (AR) time series models have various practical applications. For example, Resnick (1997) fitted such a model to interarrival times between packet transmissions on a computer network, Gallagher (2001) studied differenced sea surface temperatures and fitted a symmetric α-stable AR model, and Ling (2005) examined the daily log-returns of the Hang Seng Index in the Hong Kong stock market. When modeling infinite variance autoregressive processes, non-Gaussian α-stable distributions (i.e. the exponent parameter $\alpha < 2$) are often adopted to specify the innovation process due to their intriguing mathematical properties. This rich class of probability distributions allows heavy tails and skewness, the features exhibited in many observed time series including signal processing in electrical engineering Stuck and Kleiner (1974); Sheng and Chen (2011), portfolio selection Rachev et al. (2004), and asset allocation Tokat and Schwartz (2002). So, the use of α-stable AR models is well justified both theoretically and empirically.

When studying AR processes, causality (all roots of AR polynomial are outside the unit circle) is conventionally assumed. However, such an assumption is only needed when the study is carried out within the classical Gaussian framework in order to ensure the identifiability of model parameters. Indeed, for every non-causal Gaussian AR process there exists an equivalent causal representation in the sense that the two processes have the same mean and autocorrelation functions (see Brockwell and Davis (1991)). Since a Gaussian distribution is uniquely determined by its first two moments, the two processes necessarily possess the identical probability structure and hence are indistinguishable. In contrast, under a non-Gaussian setting, a non-causal AR process will have a different probability structure than its causal representation. In other words, for a non-Gaussian AR process the model parameters are identifiable and the model can be configured uniquely without being confined to the causal case; see Breidt and Davis (1992) and Rosenblatt (2000).

In this work we consider diagnostic tests for non-Gaussian non-causal α-stable AR processes. We remove the assumption of causality and refer to such processes as general AR processes. There has been a certain amount of work in the literature on general AR processes. For example, Breidt et al. (1991) discussed a maximum likelihood procedure for parameter estimation for autoregressive processes with non-Gaussian innovations. Andrews et al. (2009) studied maximum likelihood estimation for general AR processes with non-Gaussian α-stable innovations. They showed that, when fitting trading volumes of the Wal-Mart stock, a general model yielded a better description of the observed data in the sense that the residuals are more compatible with the assumption of independent innovations than the residuals produced by its causal representation. Lanne et al. (2010) considered forecasting of the non-causal AR time series and demonstrated the improvements in the change-of-direction forecasts when relaxing causality in the AR model fitted to the US inflation series. Recently Andrews and Davis (2011) developed a procedure of model identification for infinite variance AR processes and showed that minimizing Gaussian-based AIC yields a consistent estimator of the AR order.

Compared to the devotions received to parameter estimation and model identification for infinite variance non-causal AR processes, model diagnostics have not been fully addressed so far. This work intends to fill the gap. Utilizing the recent results of Lee and Ng (2010) and Bouhaddioui and Ghoudi (2012) we develop portmanteau test procedures for checking the goodness-of-fit of the non-causal α-stable AR model, where the model parameters are fit using maximum likelihood estimation. As second moments do not exist for infinite variance models, the behavior of the sample autocorrelation of the residuals from the fitted model is hard to harness for the purpose of model diagnostics. To circumvent the difficulty, we propose to use the trimmed residuals or nonparametric procedures based on the ranks of the residuals or the squared residuals. We show that the sample autocorrelation of trimmed residuals at a given lag for fitted general AR processes is asymptotically normal and hence the commonly used portmanteau tests in the classical Gaussian framework that are based on sample ACF, such as Box and Pierce (1970) and Ljung and Box (1978), can be easily extended to an infinite variance setting. We also proved that the rank correlations of the residuals or the squared residuals are asymptotically normal. Thus nonparametric tests could also be developed for model diagnostic purpose.

The rest of the paper is organized as follows. In section 2, we introduce the necessary background material to derive the asymptotic distribution of trimmed residuals. We then discuss the use of nonparametric and propose nonparametric methods.Using the asymptotic properties we propose an assortment of portmanteau test based on the classical methods and recent results. In section 3, we examine the finite sample performance of the proposed procedures through simulation studies. We check and compare the empirical sizes and powers of the tests. All technical proofs are relegated to the Appendix.

2 Theoretical Results

2.1 Preliminaries

Let $\{Y_t\}$ be the autoregressive process satisfying the stochastic difference equation

$$\phi(B)Y_t = Z_t, \tag{1}$$

where the AR characteristic polynomial has no zeros on the units circle, $\phi(z) := 1 - \phi_1 z - \cdots - \phi_p z^p \neq 0$ for $|z| = 1$, and the i.i.d innovation variables $\{Z_t\}$ have a stable distribution with exponent $\alpha \in (0, 2)$. We also assume that the AR characteristic polynomial could be written as the product of causal and purely non-causal polynomials,

$$\phi(z) = (1 - \theta_1 z - \cdots - \theta_r z^r)(1 - \theta_{r+1} z - \cdots - \theta_{r+s} z^s).$$

Then the unique strictly stationary solution to (1) is given by $Y_t = \sum_{j=-\infty}^{\infty} \psi_j Z_{t-j}$, where ψ_j's are determined by the Laurent series expansion for $1/\phi(z)$, $1/\phi(z) = \sum_{j=-\infty}^{\infty} \psi_j z^j$. It is well known that the coefficients $\{\psi_j\}$ are geometrically decaying; namely there exist $C_1 > 0$ and $0 < D_1 < 1$ such that $|\psi_j| < C_1 D_1^{|j|}$ for all j. Now let $\bar{\psi}_j = \psi_{-j}$, for $j > 0$. We rewrite the solution to (1) as

$$Y_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j} + \sum_{j=1}^{\infty} \bar{\psi}_j Z_{t+j}. \tag{2}$$

For the AR model defined in (1), let $\hat{\phi}$ be the MLE estimator by Andrews et al. (2009). Then $n^{1/\alpha}(\phi - \hat{\phi}) \xrightarrow{L} S$, where S is some random variable. It could be shown that there exists δ, satisfying $2\alpha/(2+\alpha) < \delta < \min(\alpha, 1)$, such that $n^{-1/\alpha} = o(n^{-1/\delta + 1/2})$.

Suppose the observed time series is represented as $\{Y_{-p+1}, \cdots, Y_0, Y_1, \cdots, Y_n\}$. Then the residuals of the fitted model, $\{\hat{Z}_t\}_{t=1}^n$, are given by

$$\hat{Z}_t = Y_t - \hat{\phi}_1 Y_{t-1} - \cdots - \hat{\phi}_p Y_{t-p}. \tag{3}$$

Let $\{\hat{Z}_t\}_{t=1}^n$ be the residuals of the fitted model. For some predetermined lower percentile λ^L and upper percentile λ^U, let $\hat{M}_n^L$ and $\hat{M}_n^U$ be the $(n\lambda^L)$-th and $(n\lambda^U)$-th order statistics of $\{\hat{Z}_t\}_{t=1}^n$, respectively. We define the following trimmed residuals

$$\hat{\tau}_t = \hat{Z}_t I_{(\hat{M}_n^L < \hat{Z}_t < \hat{M}_n^U)}.$$

The goal is to test the hypotheses where the null (H_0) is that the ARMA model

(1) with $s > 0$ is adequately identified. For the trimmed residuals, the sample autocorrelation at lag k, $\hat{\rho}_k$, is computed by the formula

$$\hat{\rho}_k = \frac{\left(\sum_{t=k+1}^{n} \hat{\tau}_t \hat{\tau}_{t-k}\right) - \left(\sum_{t=k+1}^{n} \hat{\tau}_t\right)\left(\sum_{t=k+1}^{n} \hat{\tau}_{t-k}\right)/(n-k)}{\left(\sum_{t=1}^{n} \hat{\tau}_t^2\right) - \left(\sum_{t=1}^{n} \hat{\tau}_t\right)^2/n}. \tag{4}$$

Theorem 1. *If the model (1) is correctly identified by the MLE method, then, for any positive integer m, we have*

$$\sqrt{n}\hat{\boldsymbol{\rho}}_{(m)} \xrightarrow{D} N(0, \mathbf{I}_m),$$

where $\hat{\boldsymbol{\rho}}_{(m)} := (\hat{\rho}_1, \ldots, \hat{\rho}_m)^T$ and $\mathbf{I}_m$ is the $m \times m$ identity matrix

The sample partial autocorrelation (PACF) at lag k, $\hat{\pi}_k$, can be derived by Durbin–Levison algorithm:

$$\hat{\pi}_k = \frac{\hat{\rho}_k - \hat{\boldsymbol{\rho}}_{(k-1)}^T \mathbf{R}_{(k-1)}^{-1} \hat{\boldsymbol{\rho}}_{(k-1)}^*}{1 - \hat{\boldsymbol{\rho}}_{(k-1)}^T \mathbf{R}_{(k-1)}^{-1} \hat{\boldsymbol{\rho}}_{(k-1)}}, \tag{5}$$

where $\hat{\boldsymbol{\rho}}_{(k-1)} = (\hat{\rho}_1, \ldots, \hat{\rho}_{k-1})^T$, $\mathbf{R}_{(k-1)} = (\hat{\rho}_{|i-j|})_{i,j=1}^k$ (i.e. the symmetric Toeplitz matrix generated by $(1, \hat{\rho}_1, \ldots, \hat{\rho}_{k-1})$), and $\hat{\boldsymbol{\rho}}_{(k-1)}^* = (\hat{\rho}_{k-1}, \ldots, \hat{\rho}_1)^T$.

Theorem 2. *If the model (1) is correctly identified by the MLE method, then, for any positive integer m, we have*

$$\sqrt{n}\hat{\boldsymbol{\pi}}_{(m)} \xrightarrow{D} N(0, \mathbf{I}_m),$$

where $\hat{\boldsymbol{\pi}}_{(m)} := (\hat{\pi}_1, \ldots, \hat{\pi}_m)^T$ and $\mathbf{I}_m$ is the $m \times m$ identity matrix

Nonparametric portmanteau tests could also be developed. The following result provides the foundation for nonparametric tests based on the empirical process of the residuals or the squared residuals. Let $\tilde{r}_j = \sum_{i=1}^{n} \mathrm{I}\{\hat{Z}_i \leq \hat{Z}_j\}/n$ be the normalized rank of $\hat{Z}_j$ and define the rank correlation as $\hat{\gamma}_i = \sum_{t=1}^{n-i}(\tilde{r}_t - 1/2)(\tilde{r}_{t+i} - 1/2)$. We can also define the rank correlations for the squared residuals, $\hat{\gamma}_i^*$, in the same fashion.

Theorem 3. *If the model (1) is correctly identified by MLE method, then, for any positive integer m, we have*

$$12\sqrt{n}\hat{\boldsymbol{\gamma}}_{(m)} \xrightarrow{D} N(0, \mathbf{I}_m)$$
$$12\sqrt{n}\hat{\boldsymbol{\gamma}}^*_{(m)} \xrightarrow{D} N(0, \mathbf{I}_m)$$

where $\hat{\boldsymbol{\gamma}}_{(m)} := (\hat{\gamma}_1, \ldots, \hat{\gamma}_m)^T$, $\hat{\boldsymbol{\gamma}}^*_{(m)} := (\hat{\gamma}^*_1, \ldots, \hat{\gamma}^*_m)^T$ *and* $\mathbf{I}_m$ *is the* $m \times m$ *identity matrix.*

□

2.2 Goodness-of-fit testing

The results of Theorems 1, 2 and 3 allow for the construction of the so called Portmanteau Statistics for time series goodness-of-fit. A Box and Pierce (1970) or Ljung and Box (1978) type statistic can be constructed, consider:

$$Q_{\ell b}(m) = n(n+2)\sum_{k=1}^{m}\frac{\hat{\rho}_k}{n-k}. \tag{6}$$

Under the null hypothesis, the Ljung Box type statistic will behave as a chi-square random variable with m degrees of freedom.

A statistic inspired by Monti (1994) can be constructed utilizing the partial autocorrelation function of trimmed residuals and Theorem 2,

$$Q_{mt}(m) = n(n+2)\sum_{k=1}^{m}\frac{\hat{\pi}_k^2}{n-k} \tag{7}$$

will be asymptotically distributed as a chi-square random variable with m degrees of freedom for a given positive integer m.

Recent work in the literature has suggested asymmetric statistics may be more powerful in some situations than the symmetric (i.e. equally weighted) Ljung Box and Monti type statistics. Define $\hat{R}_m$ as the Toeplitz matrix of autocorrelations:

$$\hat{R}_m = \begin{bmatrix} 1 & \hat{\rho}_1 & \dots & \hat{\rho}_m \\ \hat{\rho}_1 & 1 & \dots & \hat{\rho}_{m-1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\rho}_m & \hat{\rho}_{m-1} & \dots & 1 \end{bmatrix}.$$

Peña and Rodríguez (2002) suggest a statistics based on the likelihood ratio test from multivariate analysis. Their statistic is $\hat{D} = n(1 - |\hat{R}_m|^{1/m})$. Utilizing the asymptotic normality from Theorem 2 and an application of the delta-method, the asymptotic distribution under the null hypothesis can be shown to satisfy

$$\hat{D} \xrightarrow{D} \sum_{k=1}^{m} \frac{m-k+1}{m} \chi_k^2 \tag{8}$$

where each χ_k^2 is a chi-square random variable with one degree of freedom. This distribution is difficult to write explicitly but can be well approximated by a Gamma distribution; see Peña and Rodríguez (2002) for details.

In Peña and Rodríguez (2006) they suggest the sum of the log of one minus the squared partial autocorrelation function. Utilizing Theorem 2, that statistic can also be shown to satisfy (8). Mahdi and McLeod (2012) generalize the result of Peña and Rodríguez (2002, 2006) to the multivariate time series setting. In the univariate case their statistic is

$$Q_{gv}(m) = \frac{-3n}{2m+1} \log |\hat{R}_m| \tag{9}$$

and the distribution follows a result similar to (8) and can be approximated with a chi-square with $(3/2)m(m+1)/(2m+1)$ degrees of freedom.

Recently, Fisher and Gallagher (2012) suggest an alternative asymmetric test compared to those based on the determinant of the matrix $\hat{R}_m$. They suggest a

Weighted Ljung Box

$$Q_{w\ell}(m) = n(n+2)\sum_{k=1}^{m}\frac{m-k+1}{m}\frac{\hat{\rho}_k^2}{n-k} \tag{10}$$

which is shown to satisfy the distribution in (8) and can be well approximated by a Gamma random variable with shape $\alpha = 3m(m+1)/(8m+4)$ and scale $\beta = 2(2m+1)/3m$. Likewise, a Weighted Monti statistic is also introduced that follows the same asymptotic distribution under the null hypothesis.

3 Simulation Studies

Computation on α-stable distributions has been well studied and is known to be computationally difficult. Our studies were performed in the GNU-licensed R-Project utilizing the stable distribution in the `stabledist` package with parameterization method zero. Due to the computational intensity in optimizing the likelihood function in Andrews et al. (2009), much of our studies were run in a parallel framework utilizing the `multicore` package. Similar to Andrews et al. (2009), when optimizing the likelihood function we generate 1200 random initial conditions; the likelihood function is found for each, and then the Nelder-Mead optimization routine is run on the best eight. The parameters for maximum likelihood function of those eight is chosen as the MLE for the general AR process with α-stable innovations. Since the maximum likelihood function is found we can easily calculate the model identification criterion from Andrews and Davis (2011) as well.

In our studies we compare the Ljung Box type statistic in (6), the Monti type in (7), the Mahdi McLeod type in (9), the corresponding Weighted version of Box-Pierce Q_{wb}, Monti bype Q_{wm}, and Ljung Box test $Q_{w\ell}$ in (10) and the nonparametric test $Q_{rk} = 144n\sum_{k=1}^{m}\hat{\gamma}_k^2$ for the residuals and $Q_{rks} = 144n\sum_{k=1}^{m}(\hat{\gamma}_k^*)^2$ for the squared residuals. The Mahdi McLeod was chosen over the suggestions in Peña and Rodríguez

(2002, 2006) since it is numerically stable (see Lin and McLeod (2006)), has conservative Type I error performance and is implemented in the `portes` package. The statistics from Fisher and Gallagher (2012) are available in the `WeightedPortTest` package and include unweighted versions as well; i.e. the traditional Ljung Box and Monti types. When trimming the residuals, we truncate at the first and 99th percentiles.

We check the finite sample sizes and powers of the proposed tests for different AR(1) and AR(2) models. For each selected model the simulation is run 1000 times. The results are summarized in Table 1 through Table 3. Overall, these tests perform well when $\alpha \geq 1.5$. No test dominate the performance. As α decreases the empirical sizes increase. But since in practice the α for the fitted model is always above 1.5, the problem does not cause big concerns to us.

4 Appendix

To prove Theorem 1, we follow the method used in Lee and Ng (2010). Since Proposition 5.2 is true for the innovation process in general, we can use it for free. The key is to establish the remaining technical lemmas in their paper for the non-causal model. In the following, Proposition 1 and 2 are corresponding to Proposition 5.1 and 5.3 of Lee and Ng respectively.

Let $\varphi_t = Z_t - \hat{Z}_t$, for $t = 1, \cdots, n$. From (2) we get

$$\varphi_t = \sum_{j=1}^{p}(\phi_j - \hat{\phi}_j)Y_{t-j} = \sum_{j=1}^{p}(\phi_j - \hat{\phi}_j)\sum_{k=0}^{\infty}\psi_k Z_{t-j-k} + \sum_{j=1}^{p}(\phi_j - \hat{\phi}_j)\sum_{k=1}^{\infty}\bar{\psi}_k Z_{t-j+k} \quad (11)$$

By changing the order of summation

Table 1: Empirical sizes of non-causal AR(2) model with s=1, n=500

AR(2) $\phi_1 = 2.8$	$\phi_2 = -1.6$	$Q_{\ell b}$	Q_{mt}	Q_{rk}	Q_{gv}	Q_{rks}	Q_{wb}	$Q_{w\ell}$	Q_{wm}
$\alpha = 1.8$	5	0.042	0.045	0.030	0.020	0.051	0.034	0.036	0.037
$\beta = 0$	10	0.050	0.049	0.035	0.020	0.034	0.035	0.039	0.040
$\gamma = 1$	15	0.045	0.049	0.035	0.018	0.039	0.046	0.050	0.046
$\delta = 0$	20	0.043	0.045	0.034	0.017	0.039	0.049	0.056	0.049
	25	0.042	0.054	0.038	0.014	0.047	0.039	0.052	0.048
$\alpha = 1.5$	5	0.032	0.029	0.030	0.031	0.043	0.030	0.031	0.034
$\beta = 0$	10	0.044	0.046	0.032	0.023	0.041	0.032	0.034	0.036
$\gamma = 1$	15	0.045	0.044	0.024	0.020	0.038	0.036	0.039	0.042
$\delta = 0$	20	0.048	0.048	0.039	0.016	0.041	0.038	0.042	0.042
	25	0.039	0.046	0.037	0.015	0.046	0.034	0.043	0.047
$\alpha = 1.2$	5	0.059	0.062	0.059	0.037	0.059	0.052	0.050	0.053
$\beta = 0$	10	0.060	0.056	0.062	0.029	0.062	0.057	0.061	0.056
$\gamma = 1$	15	0.056	0.055	0.059	0.025	0.052	0.057	0.062	0.058
$\delta = 0$	20	0.049	0.053	0.067	0.021	0.057	0.057	0.060	0.056
	25	0.050	0.062	0.061	0.018	0.059	0.054	0.059	0.059
$\alpha = 0.8$	5	0.088	0.083	0.073	0.068	0.087	0.084	0.087	0.086
$\beta = 0$	10	0.078	0.081	0.085	0.053	0.069	0.088	0.091	0.088
$\gamma = 1$	15	0.081	0.084	0.078	0.047	0.071	0.087	0.091	0.092
$\delta = 0$	20	0.086	0.080	0.074	0.048	0.068	0.079	0.085	0.084
	25	0.083	0.078	0.071	0.046	0.063	0.074	0.082	0.080
$\alpha = 1.8$	5	0.045	0.045	0.038	0.022	0.041	0.030	0.030	0.029
$\beta = 0.5$	10	0.046	0.047	0.034	0.017	0.039	0.043	0.044	0.044
$\gamma = 1$	15	0.042	0.040	0.040	0.019	0.040	0.041	0.043	0.042
$\delta = 0$	20	0.046	0.039	0.037	0.014	0.046	0.037	0.042	0.041
	25	0.045	0.044	0.035	0.015	0.044	0.033	0.040	0.039
$\alpha = 1.5$	5	0.048	0.043	0.037	0.036	0.044	0.039	0.041	0.042
$\beta = 0.5$	10	0.038	0.043	0.031	0.032	0.033	0.034	0.038	0.037
$\gamma = 1$	15	0.042	0.040	0.042	0.034	0.035	0.038	0.042	0.034
$\delta = 0$	20	0.036	0.044	0.035	0.034	0.034	0.035	0.036	0.036
	25	0.035	0.033	0.030	0.034	0.036	0.031	0.037	0.036
$\alpha = 1.2$	5	0.066	0.063	0.047	0.054	0.069	0.062	0.064	0.060
$\beta = 0.5$	10	0.061	0.066	0.045	0.066	0.064	0.062	0.064	0.061
$\gamma = 1$	15	0.063	0.070	0.040	0.070	0.064	0.064	0.069	0.063
$\delta = 0$	20	0.06	0.075	0.053	0.075	0.062	0.065	0.067	0.065
	25	0.06	0.061	0.054	0.076	0.056	0.065	0.068	0.064
$\alpha = 0.8$	5	0.099	0.104	0.098	0.043	0.094	0.099	0.100	0.102
$\beta = 0.5$	10	0.114	0.113	0.091	0.047	0.083	0.115	0.117	0.115
$\gamma = 1$	15	0.115	0.110	0.098	0.053	0.079	0.104	0.109	0.115
$\delta = 0$	20	0.105	0.102	0.094	0.059	0.073	0.103	0.110	0.109
	25	0.108	0.115	0.087	0.060	0.071	0.095	0.105	0.109

Table 2: Empirical powers for non-causal AR(2) model fitted as non-causal AR(1), n=50

AR(2) $\phi_1 = -1.2$	$\phi_2 = 1.6$	$Q_{\ell b}$	Q_{mt}	Q_{gv}	Q_{wb}	Q_{rk}
$\alpha = 0.8$	5	0.616	0.541	0.67	0.756	0.915
$\beta = 0$	10	0.483	0.325	0.523	0.629	0.833
$\gamma = 1$	15	0.397	0.246	0.420	0.574	0.752
$\delta = 0$	20	0.373	0.167	0.326	0.538	0.693
	25	0.356	0.135	0.258	0.504	0.623
$\alpha = 1.2$	5	0.533	0.454	0.550	0.633	0.684
$\beta = 0$	10	0.425	0.311	0.446	0.556	0.550
$\gamma = 1$	15	0.381	0.219	0.349	0.509	0.462
$\delta = 0$	20	0.376	0.167	0.276	0.487	0.404
	25	0.349	0.117	0.204	0.460	0.360
$\alpha = 1.5$	5	0.426	0.358	0.428	0.497	0.456
$\beta = 0$	10	0.323	0.241	0.335	0.447	0.343
$\gamma = 1$	15	0.302	0.174	0.262	0.407	0.264
$\delta = 0$	20	0.284	0.138	0.192	0.379	0.227
	25	0.311	0.107	0.145	0.361	0.178
$\alpha = 1.8$	5	0.313	0.263	0.305	0.362	0.323
$\beta = 0$	10	0.235	0.188	0.247	0.319	0.214
$\gamma = 1$	15	0.228	0.145	0.193	0.296	0.155
$\delta = 0$	20	0.230	0.115	0.142	0.275	0.116
	25	0.246	0.092	0.094	0.282	0.095

Table 3: Empirical powers for non-causal AR(2) model fitted as non-causal AR(1), n=75

AR(2) $\phi_1 = -1.2$	$\phi_2 = 1.6$	$Q_{\ell b}$	Q_{mt}	Q_{gv}	Q_{wb}	Q_{rk}
$\alpha = 0.8$	5	0.909	0.851	0.933	0.948	0.99
$\beta = 0$	10	0.692	0.591	0.853	0.905	0.973
$\gamma = 1$	15	0.601	0.447	0.723	0.801	0.954
$\delta = 0$	20	0.554	0.351	0.617	0.727	0.935
	25	0.492	0.296	0.547	0.687	0.900
$\alpha = 1.2$	5	0.831	0.791	0.868	0.873	0.911
$\beta = 0$	10	0.678	0.583	0.783	0.833	0.829
$\gamma = 1$	15	0.598	0.470	0.694	0.776	0.770
$\delta = 0$	20	0.568	0.377	0.608	0.732	0.729
	25	0.533	0.312	0.537	0.694	0.673
$\alpha = 1.5$	5	0.727	0.692	0.744	0.772	0.785
$\beta = 0$	10	0.606	0.528	0.689	0.742	0.657
$\gamma = 1$	15	0.537	0.422	0.617	0.687	0.580
$\delta = 0$	20	0.506	0.344	0.541	0.658	0.522
	25	0.492	0.277	0.469	0.635	0.464
$\alpha = 1.8$	5	0.554	0.519	0.586	0.618	0.628
$\beta = 0$	10	0.470	0.420	0.521	0.586	0.485
$\gamma = 1$	15	0.421	0.339	0.470	0.545	0.394
$\delta = 0$	20	0.413	0.285	0.408	0.522	0.343
	25	0.389	0.240	0.343	0.507	0.288

$$
\begin{aligned}
|\sum_{j=1}^{p}(\phi_j - \hat{\phi}_j)\sum_{k=0}^{\infty}\psi_k Z_{t-j-k}| &\le |\sum_{j=1}^{\infty}\sum_{k=1}^{\min(j,p)}(\phi_k - \hat{\phi}_k)\psi_{j-k}Z_{t-j}| \\
&\le \sum_{j=1}^{\infty}\sum_{k=1}^{\min(j,p)}\left\|\phi - \hat{\phi}\right\||\psi_{j-k}||Z_{t-j}|, \qquad (12)
\end{aligned}
$$

and

$$
\begin{aligned}
&|\sum_{j=1}^{p}(\phi_j - \hat{\phi}_j)\sum_{k=1}^{\infty}\bar{\psi}_k Z_{t-j+k}| \\
&\le \sum_{j=0}^{\infty}|\sum_{k=1}^{p}(\phi_k - \hat{\phi}_k)\bar{\psi}_{j+k}Z_{t+j}| + |\sum_{j=1}^{p-1}\sum_{k=j+1}^{p}(\phi_k - \hat{\phi}_k)\bar{\psi}_{k-j}Z_{t-j}| \\
&\le \sum_{j=0}^{\infty}\sum_{k=1}^{p}\left\|\phi - \hat{\phi}\right\||\bar{\psi}_{j+k}Z_{t+j}| + \sum_{j=1}^{p-1}\sum_{k=j+1}^{p}\left\|\phi - \hat{\phi}\right\||\bar{\psi}_{k-j}Z_{t-j}|, \qquad (13)
\end{aligned}
$$

where $\left\|\phi - \hat{\phi}\right\|$ is the Euclidean distance of ϕ and $\hat{\phi}$. By Andrews and Davis (2009) the MLE estimator of the AR polynomial coefficients, $\hat{\phi}$, converges to some random variable in distribution $n^{1/\alpha}(\hat{\phi} - \phi) \xrightarrow{D} S$. By our assumption that $0 < \alpha < 2$, we can find a δ with $2\alpha/(\alpha + 2) < \delta < min\{\alpha, 1\}$ such that $n^{-\alpha} = o(n^{-1/\delta+1/2})$. Note that for any given $\epsilon > 0$ there always exists a $\gamma_1 > 0$ such that $\mathrm{P}(|S| > \gamma_1) < \epsilon/2$. If we define define $A_n = \left\{\left\|\phi - \hat{\phi}\right\| < \gamma_1 n^{-1/\alpha}\right\}$ then there exists $N > 0$ such that $P(A_n) > 1 - \epsilon$ whenever $n > N_1$. Under the condition of A_n, we can obtain an upper bound for (12)

$$
|\sum_{j=1}^{p}(\phi_j - \hat{\phi}_j)\sum_{j=0}^{\infty}\psi_k Z_{t-k}| \le \gamma_1 n^{-1/\alpha}\sum_{j=1}^{\infty}\sum_{k=1}^{\min(j,p)}|\psi_{j-k}||Z_{t-j}|, \qquad (14)
$$

and an upper bound for (13)

$$|\sum_{j=1}^{p}(\phi_j-\hat{\phi}_j)\sum_{k=1}^{\infty}\bar{\psi}_k Z_{t+k}| \le \gamma_1 n^{-1/\alpha}\sum_{j=0}^{\infty}\sum_{k=1}^{p}|\bar{\psi}_{j+k}||Z_{t+j}| + \gamma_1 n^{-1/\alpha}\sum_{j=1}^{p-1}\sum_{k=j+1}^{p}|\bar{\psi}_{k-j}||Z_{t-j}|. \tag{15}$$

Let $\psi_j^* = \sum_{k=1}^{\min(j,p)} |\psi_{j-k}|$, $\bar{\psi}_j^* = \sum_{k=1}^{p} |\bar{\psi}_{j+k}|$, and $\bar{\psi}_{1,\cdots,p} = \sum_{k=1}^{p} |\bar{\psi}_k|$.

Assuming A_n is true, an upper bound for $|\varphi_t|$ is given by

$$|\varphi_t| \le \gamma_1 n^{-1/\alpha}\sum_{j=1}^{\infty}\psi_j^*|Z_{t-j}| + \gamma_1 n^{-1/\alpha}\sum_{j=0}^{\infty}\bar{\psi}_j^*|Z_{t+j}| + \gamma_1 n^{-1/\alpha}\sum_{j=1}^{p-1}\bar{\psi}_{1,\cdots,p}|Z_{t-j}|. \tag{16}$$

Proposition 1. *For (16), the following are true,*

$$E|\sum_{j=1}^{\infty}\psi_j^*|Z_{t-j}||^{\delta} < \infty,$$

$$E|\sum_{j=0}^{\infty}\bar{\psi}_j^*|Z_{t+j}||^{\delta} < \infty,$$

$$\gamma_1^{\delta} n^{-\delta/\alpha}\sum_{t=1}^{n}\{E|\sum_{j=1}^{\infty}\psi_j^*|Z_{t-j}||^{\delta} + E|\sum_{j=0}^{\infty}\bar{\psi}_j^*|Z_{t+j}||^{\delta}\} = o(n).$$

Proof. The coefficients $\{\psi_j\}$ and $\{\bar{\psi}_j\}$ are geometrically decaying as $j \to \infty$. As a result, $\sum_{j=1}^{\infty}|\psi_j|^{\delta} < \infty$ and $\sum_{j=1}^{\infty}|\psi_j^*|^{\delta} < \infty$.

Change the order of summation and apply the triangle inequality, then we have

$$\sum_{j=1}^{\infty}|\psi_j^*|^{\delta} \le \sum_{j=1}^{\infty}\sum_{k=1}^{\min(j,p)}|\psi_{j-k}|^{\delta} = p\sum_{j=0}^{\infty}|\psi_j|^{\delta} < \infty,$$

and

$$\sum_{j=0}^{\infty}|\bar{\psi}_j{}^*|^{\delta} \le \sum_{j=0}^{\infty}\sum_{k=1}^{p}|\bar{\psi}_{j+k}|^{\delta} \le p\sum_{j=1}^{\infty}|\bar{\psi}_j|^{\delta} < \infty.$$

Also by the triangle inequality (for example, page 537, Brockwell and Davis, 1991) and $E|Z_{t-j}|^{\delta} < \infty$

$$\begin{aligned} E|\sum_{j=1}^{\infty} \psi_j^* |Z_{t-j}||^{\delta} &\leq E|\sum_{j=1}^{\infty} |\psi_j^*|^{\delta} ||Z_{t-j}|^{\delta} < \infty, \\ E|\sum_{j=0}^{\infty} \bar{\psi}_j^* |Z_{t+j}||^{\delta} &\leq E|\sum_{j=0}^{\infty} |\bar{\psi}_j^*|^{\delta} ||Z_{t+j}|^{\delta} < \infty. \end{aligned}$$

$\square$

Given a fixed number $0 < \lambda < 1$ and β_n a predetermined sequence of real numbers, let $\chi_t = Z_t - Z_{([n\lambda])} - \beta_n$. The Proposition 5.3. of Lee and Ng (2010) is also true for the non-causal AR sequences.

Proposition 2. *For any* $\gamma_2 > 0$,

$$\mathrm{P}\{n^{-1/2} \sum_{t=1}^{n} 1_{(|\varphi_t| > |\chi_t|)} 1_{A_n} > \gamma_2\} \to 0.$$

Proof. As in Lee and Ng (2010), we can pick a constant $\gamma_3 > 0$ such that $\mathrm{P}(|Z_{s[n\lambda]}| > \gamma_3)$ is arbitrarily small in which $s(k) = j$ if Z_j is the k^{th} largest number among $\{Z_1, \ldots, Z_n\}$. To show the result it is sufficient to get

$$\sum_{t=1}^{n} \mathrm{P}\{(|\varphi_t| > |\chi_t|) \cap A_n \cap (|Z_{s[n\lambda]}| < \gamma_3)\} = o(n^{1/2}).$$

By Lee and Ng (2010), for any $t \in \{1, \ldots, n\}$,

$$\mathrm{P}\{(|\varphi_t| > |\chi_t|) \cap A_n \cap (|Z_{s[n\lambda]}| < \gamma_3)\} \leq \frac{1}{n} + \frac{n-1}{n} E\{|\varphi_t|^{\delta} |\chi_t|^{-\delta} 1_{(|Z_{s[n\lambda]}| < \gamma_3)} 1_{A_n} \Big| t \neq s([n\lambda])\}.$$

Use triangle inequality and (16)

$$\sum_{t=1}^{n} E\{|\varphi_t|^\delta |\chi_t|^{-\delta} 1_{(|Z_{s[n\lambda]}|<\gamma_3)} 1_{A_n} \Big| t \neq s([n\lambda])\}$$

$$\leq \sum_{t=1}^{n} E\{\gamma_1^\delta n^{-\delta/\alpha} (\sum_{j=1}^{\infty} \psi_j^* |Z_{t-j}|)^\delta |\chi_t|^{-\delta} 1_{(|Z_{s[n\lambda]}|<\gamma_3)} \Big| s([n\lambda]) \neq t\} + \tag{17}$$

$$\sum_{t=1}^{n} E\{\gamma_1^\delta n^{-\delta/\alpha} (\sum_{j=0}^{\infty} \bar{\psi}_j^* |Z_{t+j}|)^\delta |\chi_t|^{-\delta} 1_{(|Z_{s[n\lambda]}|<\gamma_3)} \Big| s([n\lambda]) \neq t\} + \tag{18}$$

$$\sum_{t=1}^{n} E\{\gamma_1^\delta n^{-\delta/\alpha} (\sum_{j=1}^{p-1} \bar{\psi}_{1,\cdots,p} |Z_{t-j}|)^\delta |\chi_t|^{-\delta} 1_{(|Z_{s[n\lambda]}|<\gamma_3)} \Big| s([n\lambda]) \neq t\}. \tag{19}$$

To finish the proof, in the next we will show (17), (18), and (19) are $o(n^{1/2})$. Conditional on $s([n\lambda]) = t - j$ and $s([n\lambda]) \neq t - j$, (17) is bounded above by

$$\sum_{t=1}^{n} \gamma_1^\delta \gamma_3^\delta n^{-\delta/\alpha} \sum_{j=1}^{\infty} |\psi_j^*|^\delta E\{|\chi_t|^{-\delta} \Big| s([n\lambda]) \neq t\} + \tag{20}$$

$$\sum_{t=1}^{n} E\{\gamma_1^\delta n^{-\delta/\alpha} \sum_{j=1}^{\infty} |\psi_j^*|^\delta |Z_{t-j}|^\delta |\chi_t|^{-\delta} 1_{(s[n\lambda]\neq t-j)} \Big| s([n\lambda]) \neq t\}. \tag{21}$$

We apply Proposition 1, (5.23) in Proposition 5.3 of Lee and Ng (2010), and the fact that $n^{-1/\alpha} = o(n^{-1/\delta+1/2})$ to (20) and get

$$\sum_{t=1}^{n} \gamma_1^\delta \gamma_3^\delta n^{-\delta/\alpha} \sum_{j=1}^{\infty} |\psi_j^*|^\delta E\{|\chi_t|^{-\delta} \Big| s([n\lambda]) \neq t\} = o(n^{1/2}).$$

For (21), two cases, $1 \leq j \leq t-1$ and $j \geq t$, are considered respectively. When $1 \leq j \leq t-1$,

$$\sum_{t=1}^{n} E\{\gamma_1^\delta n^{-\delta/\alpha} \sum_{j=1}^{t-1} |\psi_j^*|^\delta |Z_{t-j}|^\delta |\chi_t|^{-\delta} 1_{(s[n\lambda]\neq t-j)} \Big| s([n\lambda]) \neq t\} \leq$$

$$\sum_{t=1}^{n} \gamma_1^\delta n^{-\delta/\alpha} \frac{n-2}{n-1} E\{\sum_{j=1}^{t-1} |\psi_j^*|^\delta |Z_{t-j}|^\delta |\chi_t|^{-\delta} \Big| s([n\lambda]) \neq t, t-j\} = o\left(n^{1/2}\right),$$

since by (5.22) of Lee and Ng (2010) $E\{\sum_{j=1}^{t-1} |\psi_j^*|^\delta |Z_{t-j}|^\delta |\chi_t|^{-\delta} \Big| s([n\lambda]) \neq t, t-j\} = O(1)$. When $j \geq t$, Z_{t-j} is in the set $\{Z_0, Z_{-1}, \ldots\}$. Hence, Z_{t-j} is independent of $s([n\lambda])$, which implies that

$$E\{|\psi_j^*|^\delta |Z_{t-j}|^\delta |\chi_t|^{-\delta} 1_{(s[n\lambda]\neq t-j)} \Big| s([n\lambda]) \neq t\} = |\psi_j^*|^\delta E\{|Z_{t-j}|^\delta\} E\{|\chi_t|^{-\delta} 1_{(s[n\lambda]\neq t-j)} \Big| s([n\lambda]) \neq t\}.$$

Now use Proposition 1 and (5.23) of Lee and Ng again

$$\sum_{t=1}^{n} E\{\gamma_1^\delta n^{-\delta/\alpha} \sum_{j=t}^{\infty} |\psi_j^*|^\delta |Z_{t-j}|^\delta |\chi_t|^{-\delta} 1_{(s[n\lambda]\neq t-j)} \Big| s([n\lambda]) \neq t\} \leq$$
$$\sum_{t=1}^{n} \gamma_1^\delta n^{-\delta/\alpha} \frac{n-1}{n} \sum_{j=t}^{\infty} |\psi_j^*|^\delta E\{|Z_{t-j}|^\delta\} E\{|\chi_t|^{-\delta} 1_{(s[n\lambda]\neq t-j)} \Big| s([n\lambda]) \neq t\} = o\left(n^{1/2}\right).$$

Therefore, (17) is $o(n^{1/2})$. In the same way, the result also holds for (18) and (19).

□

Proof. of Theorem 1.

Let q^L and q^U be the (λ^L)-th and (λ^U)-th quantiles of Z_t. Denote the mean and standard deviation of the trimmed random variable $Z_t I(q^L < Z_t < q^U)$ by μ and σ,

$$\mu = E[Z_t I(q^L < Z_t < q^U)] \quad \text{and} \quad \sigma^2 = Var[Z_t I(q^L < Z_t < q^U)].$$

Let $Z_t^\mu = Z_t I_t - \mu$, then directly from Lemma 4.1 in Lee and Ng (2010),

$$\begin{aligned} n^{-1/2} \left\{ \sum_{t=k+1}^{n} Z_t^\mu Z_{t-k}^\mu \right\}_{k=1,2,\ldots,m} &\xrightarrow{D} N(0, \sigma^4 I_m), \\ n^{-1/2} \sum_{t=1}^{n} Z_t^\mu \xrightarrow{D} N(0, \kappa^2), & \\ n^{-1} \sum_{t=1}^{n} (Z_t^\mu)^2 \xrightarrow{p} \sigma^2, & \qquad (22) \end{aligned}$$

with κ being certain constant associated with the distribution of Z_t, and (q^L, q^U).

Now let M_n^L and M_n^U be the $(n\lambda^L)$-th and $(n\lambda^U)$-th order statistics of $\{Z_t\}_{t=1}^n$ and define $\hat{Z}_t^\mu = Z_t I_t - \mu$ and

$$I_t = \begin{cases} 1, & \text{if } M_n^L < Z_t < M_n^U, \\ 0, & \text{otherwise,} \end{cases} \qquad \hat{I}_t = \begin{cases} 1, & \text{if } \hat{M}_n^L < \hat{Z}_t < \hat{M}_n^U, \\ 0, & \text{otherwise.} \end{cases}$$

It follows from Proposition (1), Proposition (2), and the proof of Lemma 4.2 in Lee and Ng (2010) that

$$\begin{aligned} & n^{-1/2} \sum_{t=k+1}^{n} |Z_t^\mu Z_{t-k}^\mu - \hat{Z}_t^\mu \hat{Z}_{t-k}^\mu| \xrightarrow{p} 0, \quad \text{for } k = 1, 2, \ldots, m, \\ & n^{-1/2} \sum_{t=1}^{n} |Z_t I_t - \hat{Z}_t \hat{I}_t| \xrightarrow{p} 0, \\ & n^{-1} \sum_{t=1}^{n} |(Z_t^\mu)^2 - (\hat{Z}_t^\mu)^2| \xrightarrow{p} 0. \end{aligned} \tag{23}$$

Now note that

$$\sqrt{n-k}\hat{\rho}_k = \frac{\frac{1}{\sqrt{n-k}} \sum_{t=k+1}^{n} \hat{Z}_t^\mu \hat{Z}_{t-k}^\mu - \frac{1}{\sqrt{n-k}} \sum_{t=k+1}^{n} \hat{Z}_t^\mu \frac{1}{n-k} \sum_{t=k+1}^{n} \hat{Z}_{t-k}^\mu}{\frac{1}{n} \sum_{t=1}^{n} (\hat{Z}_t^\mu)^2 - \frac{1}{n^2} (\sum_{t=1}^{n} \hat{Z}_t^\mu)^2}.$$

Combining (22) and (23) yields the result.

□

Proof. of Theorem 2.

By Theorem 1 and equation (5).

□

Proof. of Theorem 3

Define the empirical copula of the residuals be defines as

$$C_{m,n}(u_1,\ldots,u_m)=\frac{1}{\sqrt{n-m+1}}\sum_{i=1}^{n-m+1}\left[\prod_{j=1}^{m}\mathrm{I}(\tilde{r}_{i+j-1}\leq u_j)-\prod_{j=1}^{m}u_j\right],$$

where $(u_1,\ldots,u_m)\in[0,1]^m$. Cline and Brockwell (1985) showed

$$\lim_{t\to\infty}\frac{P[|Y_1|>t]}{P[|Z_1|>t]}=\sum_{j=-\infty}^{\infty}|\psi_j|^{\alpha}. \tag{24}$$

As a result

$$\lim_{t\to\infty}nP[|Y_1|>a_nt]=\sum_{j=-\infty}^{\infty}|\psi_j|^{\alpha}t^{-\alpha}, \tag{25}$$

for all $t>0$, where $a_n=inf\{t:nP[|Z_1|>t]\leq 1\}$. Then we can follow the same lines of Theorem 3.4 and related technical Lemmas in Bouhaddioui and Ghoudi (2012) to prove that the empirical copula of the residuals $C_{m,n}$ converges to a continuous process $\tilde{\mathcal{C}}$ if the model (1) is correctly identified by MLE method. The continuous process $\tilde{\mathcal{C}}$ is the limit of the sequential empirical process of a sequence of i.i.d random variables identified in Genest and Rémillard (2004), for which there is no simple expression. However, as in Proposition 2.1 of Genest and Rémillard (2004), the Möbius transformation of $C_{m,n}$, $\mathcal{M}$, leads to some simple results. Let A be a subset of $\{1,\ldots,m\}$ with $|A|>1$, the Möbius transformation of $C_{m,n}$ indexed by A is

$$\mathcal{M}_A(C_{m,n})=\frac{1}{\sqrt{n-m+1}}\sum_{i=1}^{n-m+1}\prod_{j\in A}\left[\mathrm{I}(\tilde{r}_{i+j-1}\leq u_j)-u_j\right].$$

Then $\mathcal{M}_A(C_{m,n})$ converge jointly to continuous centered Gaussian processes $\mathcal{M}_A(\tilde{\mathcal{C}})$ and furthermore $\mathcal{M}_A(\tilde{\mathcal{C}})$ and $\mathcal{M}_{A'}(\tilde{\mathcal{C}})$ are asymptotically independent whenever two sets $A\neq A'$. Letting $A=\{1,k+1\}$, then the serial rank correlation $\hat{\gamma}_i$ could be derived, as in Bouhaddioui and Ghoudi (2012), from the Möbius transformation of $C_{m,n}$ through

$$
\begin{aligned}
\hat{\gamma}_i &= \frac{1}{\sqrt{n}} \int \mathcal{M}_A(C_{m,n}) d\mathbf{u} \\
&= \frac{1}{\sqrt{n}} \int_0^1 \int_0^1 C_{m,n}(u_1, 1, \ldots, 1, u_{k+1}, 1, \ldots, 1) du_1 du_{k+1} \\
&= \frac{1}{n} \left[\sum_{t=1}^{n-i} (\tilde{r}_t - 1/2)(\tilde{r}_{t+k} - 1/2) \right]. \qquad (26)
\end{aligned}
$$

and $\sqrt{n}\hat{\gamma}_i$ is asymptotically normal with mean zero and variance $1/12^2$. The same result carries over to the case of the squared residuals as discussed in Bouhaddioui and Ghoudi (2012). □

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