

Hermite and Laguerre polynomials and matrix-valued stochastic

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Abstract

We extend to matrix-valued stochastic processes, some well-known relations between real-valued diffusions and classical orthogonal polynomials, along with some recent results about Lévy processes and martingale polynomials. In particular, joint semigroup densities of the eigenvalue processes of the generalized matrix-valued Ornstein-Uhlenbeck and squared Ornstein-Uhlenbeck processes are respectively expressed by means of the Hermite and Laguerre polynomials of matrix arguments. These polynomials also define martingales for the Brownian matrix and the generalized Gamma process. As an application, we derive a chaotic representation property for the eigenvalue process of the Brownian matrix.

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