On the sphericity of scaling limits of random planar quadrangulations

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Abstract

We give a new proof of a theorem by Le Gall and Paulin, showing that scaling limits of random planar quadrangulations are homeomorphic to the 2-sphere. The main geometric tool is a reinforcement of the notion of Gromov-Hausdorff convergence, called 1-regular convergence, that preserves topological properties of metric surfaces.

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