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A note on percolation on Z^d: isoperimetric profile via exponential cluster repulsion

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Abstract

We show that for all p>p_c(Z^d) percolation parameters, the probability that the cluster of the origin is finite but has at least t vertices at distance one from the infinite cluster is exponentially small in t. We use this to give a short proof of the strongest version of the important fact that the isoperimetric profile of the infinite cluster basically coincides with the profile of the original lattice. This implies, e.g., that simple random walk on the largest cluster of a finite box [-n,n]^d with high probability has L^infty-mixing time Theta(n^2), and that the heat kernel (return probability) on the infinite cluster a.s. decays like p_n(o,o)=O(n^{-d/2}). Versions of these results have been proven by Benjamini and Mossel (2003), Mathieu and Remy (2004), Barlow (2004) and Rau (2006). For general infinite graphs, we prove that anchored isoperimetric properties survive supercritical percolation, provided that the probability of the cluster of the origin being finite with large boundary decays rapidly; this is the case for a large class of graphs when p is close to 1. As an application (with the help of some entropy inequalities), we give a short conceptual proof of a theorem of Angel, Benjamini, Berger and Peres (2006): the infinite percolation cluster of a wedge in Z^3 is a.s. transient whenever the wedge itself is transient.

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