Discrete time nonlinear filters with informative observations are stable

Ramon Van Handel, *Princeton University*

Abstract
The nonlinear filter associated with the discrete time signal-observation model (X_k,Y_k) is known to forget its initial condition as $k\to\infty$ regardless of the observation structure when the signal possesses sufficiently strong ergodic properties. Conversely, it stands to reason that if the observations are sufficiently informative, then the nonlinear filter should forget its initial condition regardless of any properties of the signal. We show that for observations of additive type $Y_k=h(X_k)+xi_k$ with invertible observation function h (under mild regularity assumptions on h and on the distribution of the noise xi_k), the filter is indeed stable in a weak sense without any assumptions at all on the signal process. If the signal satisfies a uniform continuity assumption, weak stability can be strengthened to stability in total variation.

Full text: PDF | PostScript

Pages: 562-575

Published on: November 14, 2008

Bibliography

1. Rami Atar and Ofer Zeitouni. Exponential stability for nonlinear filtering. *Ann. Inst. H. Poincaré Probab. Statist.* 33 (1997), no. 6, 697-725. Math. Reviews 1484538 (98i:60070)
2. David Blackwell and Lester Dubins. Merging of opinions with increasing information. *Ann. Math. Statist.* 33 (1962), 882-886. Math. Reviews 0149577 (26 #7062)
3. A. Budhiraja and D. Ocone. Exponential stability in discrete-time filtering for non-ergodic signals. *Stochastic Process. Appl.* 82 (1999), no. 2, 245-257. Math. Reviews 1700008 (2000d:94010)
4. Olivier Cappé, Eric Moulines, and Tobias Rydén. *Inference in hidden Markov models.* Springer Series in Statistics, Springer, New York, 2005. With Randal Douc's contributions to Chapter 9 and Christian P. Robert's to Chapters 6, 7 and 13, With Chapter 14 by Gersende Fort, Philippe Soulier and Moulines, and Chapter 15 by Stéphane Boucheron and Elisabeth Gassiat. Math. Reviews 2159833 (2006e:60002)
5. Pavel Chigansky and Robert Liptser. Stability of nonlinear filters in nonmixing case. *Ann. Appl. Probab.* 14 (2004), no. 4, 2038-2056. Math. Reviews 2099662 (2005h:62265)
6. Pavel Chigansky and Robert Liptser. On a role of predictor in the filtering stability. *Electron. Comm. Probab.* 11 (2006), 129-140 (electronic). Math. Reviews 2240706 (2007k:60118)
7. D. Crisan and K. Heine. Stability of the discrete time filter in terms of the tails of noise distributions. *J. London Math. Soc.* 78 (2008), 441-458.
8. D. Crisan and B. Rozovsky (eds.). *The Oxford University handbook of nonlinear filtering.* Oxford University Press, 2009. To appear.
9. Pierre Del Moral and Alice Guionnet. On the stability of interacting processes with applications to filtering and genetic algorithms. *Ann. Inst. H. Poincaré Probab. Statist.* 37 (2001), no. 2, 155-194. Math. Reviews 1819122 (2002k:60013)
10. R. M. Dudley. *Real analysis and probability*. Cambridge Studies in Advanced Mathematics, vol. 74, Cambridge University Press, Cambridge, 2002. Revised reprint of the 1989 original. Math. Reviews 1932358 (2003h:60001)
11. Gerald B. Folland. *Real analysis*, second ed. John Wiley & Sons Inc., New York,

Research Support Tool

Capture Cite
View Metadata
Printer Friendly

▼ Context

Author Address

▼ Action

Email Author
Email Others

1999. Modern techniques and their applications, A Wiley-Interscience Publication. Math. Reviews 1681462 (2000c:00001)

12. Daniel Ocone and Etienne Pardoux. Asymptotic stability of the optimal filter with respect to its initial condition. *SIAM J. Control Optim.* 34 (1996), no. 1, 226-243. Math. Reviews 1372912 (97e:60073)

13. Nadia Oudjane and Sylvain Rubenthaler. Stability and uniform particle approximation of nonlinear filters in case of non ergodic signals. *Stoch. Anal. Appl.* 23 (2005), no. 3, 421-448. Math. Reviews 2140972 (2005m:93153)

14. A. N. Shiryaev. *Probability*, second ed. Graduate Texts in Mathematics, vol. 95, Springer-Verlag, New York, 1996. Translated from the first (1980) Russian edition by R. P. Boas. Math. Reviews 1368405 (97c:60003)

15. R. van Handel. The stability of conditional Markov processes and Markov chains in random environments, 2008. Preprint, http://arxiv.org/abs/0801.4366

16. R. van Handel. Uniform observability of hidden Markov models and filter stability for unstable signals, 2008. Preprint, http://arxiv.org/abs/0804.2885

17. R. van Handel. Observability and nonlinear filtering. *Probab. Th. Rel. Fields* (2009). To appear. Published electronically at http://dx.doi.org/10.1007/s00440-008-0161-y