Euler's formulae for ζ(2n) and products of Cauchy variables

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Abstract
We show how to recover Euler's formula for $\zeta(2n)$, as well as $L_{X_4}(2n+1)$, for any integer n, from the knowledge of the density of the product of independent standard Cauchy variables.

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Bibliography

1. Andrews, George E.; Askey, Richard; Roy, Ranjan. Special functions. Encyclopedia of Mathematics and its Applications, 71. *Cambridge University Press, Cambridge,* 1999. xvi+664 pp. ISBN: 0-521-62321-9; 0-521-78988-5 MR1688958 (2000g:33001)
2. Biane, Philippe; Pitman, Jim; Yor, Marc. Probability laws related to the Jacobi theta and Riemann zeta functions, and Brownian excursions. *Bull. Amer. Math. Soc. (N.S.)* 38 (2001), no. 4, 435--465 (electronic). MR1848256 (2003b:11083)
3. Chaumont, L.; Yor, M. Exercises in probability. A guided tour from measure theory to random processes, via conditioning. Cambridge Series in Statistical and Probabilistic Mathematics, 13. *Cambridge University Press, Cambridge,* 2003. xvi+236 pp. ISBN: 0-521-82585-7 MR2016344 (2004m:60001)
4. Lamperti, John. An occupation time theorem for a class of stochastic processes. *Trans. Amer. Math. Soc.* 88 1958 380--387. MR0094863 (20 #1372)
5. Lebedev, N. N. Special functions and their applications. Revised edition, translated from the Russian and edited by Richard A. Silverman. Unabridged and corrected republication. *Dover Publications, Inc., New York,* 1972. xii+308 pp. MR0350075 (50 #2568)
6. Lévy, Paul. OE uvres de Paul Lévy. Vol. VI. (French) [Works of Paul Levy. Vol. VI] Théorie des jeux. [Game theory] Published under the direction of Daniel Dugué with the collaboration of Paul Deheuvels and Michel Ibéro. *Gauthier-Villars, Paris,* 1980. 423 pp. (1 plate). ISBN: 2-04-010962-5 MR0586767 (83c:01070)
7. Pitman, Jim; Yor, Marc. Infinitely divisible laws associated with hyperbolic functions. *Canad. J. Math.* 55 (2003), no. 2, 292--330. MR1969794 (2004c:11151)
8. Pitman, Jim; Yor, Marc. Level crossings of a Cauchy process. *Ann. Probab.* 14 (1986), no. 3, 780--792. MR0841583 (87j:60106)
9. J.P. Serre, Cours d'arithmétique, *Collection SUP, P.U.F., Paris*, 1970.
10. H. M. Srivasta, Junesang Choi, Series associated with the Zeta and Related Functions, 2006, *Kluwer Academic Publishers, Dordrecht*, 2001.