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A modified Kardar--Parisi--Zhang model

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Abstract

A one dimensional stochastic differential equation of the form
$dX=A\,X\,dt+(1/2)\,(-A)^{-\alpha}\partial_\xi[((-A)^{-\alpha}X)^2]dt+\partial_\xi\,dW(t), \quad X(0)=x$
is considered, where $A=(1/2)\,\partial_\xi^2$. The equation is equipped with periodic boundary conditions. When $\alpha=0$ this equation arises in the Kardar--Parisi--Zhang model. For $\alpha\neq 0$, this equation conserves two important properties of the Kardar--Parisi--Zhang model: it contains a quadratic nonlinear term and has an explicit invariant measure which is gaussian. However, it is not as singular and using renormalization and a fixed point result we prove existence and uniqueness of a strong solution provided $\alpha>1/8$.

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