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The volume fraction of a non--overlapping germ--grain model

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Abstract

We discuss the volume fraction of a model of non--overlapping convex grains. It is obtained from thinning a Poisson process where each point has a weight and is the centre of a grain, by removing any grain that is overlapped by one of larger or equal weight. In the limit as the intensity of the Poisson process tends to infinity, the model can be identified with the intact grains in the dead leaves model if the weights are independent of the grain sizes. In this case we can show that the volume fraction is at most $1/2^d$ for $d=1$ or 2 if the shape is fixed, but the size and the orientation are random. The upper bound is achieved for centrally symmetric sets of the same size and orientation. For general d we can show the upper bound, $1/2^d$, for spherical grains with two--point radius distribution. If dependence between weight and size is allowed, it is possible to achieve a volume fraction arbitrarily close to one.

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