

A note on a.s. finiteness of perpetual integral functionals of difusions

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Abstract

In this note we use the boundary classification of diffusions in order to derive a criterion for the convergence of perpetual integral functionals of transient real-valued diffusions. We present a second approach, based on Khas'minskii's lemma, which is applicable also to spectrally negative L'evy processes. In the particular case of transient Bessel processes, our criterion agrees with the one obtained via Jeulin's convergence lemma.

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