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Asymptotic results for empirical measures of weighted sums of independent random variables

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Abstract

We investigate the asymptotic behavior of weighted sums of independent standardized random variables with uniformly bounded third moments. The sequence of weights is given by a family of rectangular matrices with uniformly small entries and approximately orthogonal rows. We prove that the empirical CDF of the resulting partial sums converges to the normal CDF with probability one. This result implies almost sure convergence of empirical periodograms, almost sure convergence of spectral distribution of circulant and reverse circulant matrices, and almost sure convergence of the CDF generated from independent random variables by independent random orthogonal matrices. In the special case of trigonometric weights, the speed of the almost sure convergence is described by a normal approximation as well as a large deviation principle.

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