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Invariant measures for stochastic Cauchy problems with asymptotically unstable drift semigroup

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Abstract

We investigate existence and permanence properties of invariant measures for abstract stochastic Cauchy problems of the form

$$dU(t) = (AU(t) + f)\,dt + B\,dW_H(t),$$

governed by the generator A of an asymptotically unstable C_0-semigroup on a Banach space E. Here f in E is fixed, W_H is a cylindrical Brownian motion over a separable real Hilbert space H, and B is a bounded operator from H to E. We show that if E does not contain a copy of c_0, such invariant measures fail to exist generically but may exist for a dense set of operators B. It turns out that many results on invariant measures which hold under the assumption of uniform exponential stability of S break down without this assumption.

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