

A Note on Occupation Times of Stationary Processes

Marina Kozlova, *Abo Akademi University, Finland*
Paavo Salminen, *Abo Akademi University, Finland*

Abstract

Consider a real valued stationary process $X = \{X_s; s \in \mathbb{R}\}$. For a fixed $t \in \mathbb{R}$ and a set D in the state space of X , let g_t and d_t denote the starting and the ending time, respectively, of an excursion from and to D (straddling t). Introduce also the occupation times $I^+_{t,D}$ and $I^-_{t,D}$ above and below, respectively, the observed level at time t during such an excursion. In this note we show that the pairs $(I^+_{t,D}, I^-_{t,D})$ and $(t-g_t, d_t-t)$ are identically distributed. This somewhat curious property is, in fact, seen to be a fairly simple consequence of the known general uniform sojourn law which implies that conditionally on $I^+_{t,D} + I^-_{t,D} = v$ the variable $I^+_{t,D}$ (and also $I^-_{t,D}$) is uniformly distributed on $(0, v)$. We also particularize to the stationary diffusion case and show, e.g., that the distribution of $I^-_{t,D} + I^+_{t,D}$ is a mixture of gamma distributions.

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Pages: 94-104

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