Almost Sure Stability of Linear Ito-Volterra Equations with Damped Stochastic Perturbations

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Abstract

In this paper we study the a.s. convergence of all solutions of the It^{o}-Volterra equation [dX(t) = (AX(t) + int_{0}^{t} K(t-s)X(s),ds),dt + Sigma(t),dW(t)] to zero. A is a constant $dtimes d$ matrix, K is a $dtimes d$ continuous and integrable matrix function, $Sigma$ is a continuous $dtimes r$ matrix function, and W is an r-dimensional Brownian motion. We show that when [x'(t) = Ax(t) + int_{0}^{t} K(t-s)x(s),ds] has a uniformly asymptotically stable zero solution, and the resolvent has a polynomial upper bound, then X converges to 0 with probability 1, provided [lim_{t rightarrow infty} |Sigma(t)|^{2}log t= 0.] A converse result under a monotonicity restriction on $|Sigma|$ establishes that the rate of decay for $|Sigma|$ above is necessary. Equations with bounded delay and neutral equations are also considered.

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