Eigenvalues of the Laguerre Process as Non-Colliding Squared Bessel Processes

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Abstract

Let $A(t)$ be an n-times-p matrix with independent standard complex Brownian entries and set $M(t)=A(t)^*A(t)$. This is a process version of the Laguerre ensemble and as such we shall refer to it as the *Laguerre process*. The purpose of this note is to remark that, assuming $n>p$, the eigenvalues of $M(t)$ evolve like p independent squared Bessel processes of dimension $2(n-p+1)$, conditioned (in the sense of Doob) never to collide. More precisely, the function $h(x)=prod_{i<j}(x_i-x_j)$ is harmonic with respect to p independent squared Bessel processes of dimension $2(n-p+1)$, and the eigenvalue process has the same law as the corresponding Doob h-transform. In the case where the entries of $A(t)$ are *real* Brownian motions, $(M(t))_{t>0}$ is the Wishart process considered by Bru (1991). There it is shown that the eigenvalues of $M(t)$ evolve according to a certain diffusion process, the generator of which is given explicitly. An interpretation in terms of non-colliding processes does not seem to be possible in this case. We also identify a class of processes (including Brownian motion, squared Bessel processes and generalised Ornstein-Uhlenbeck processes) which are all amenable to the same h-transform, and compute the corresponding transition densities and upper tail asymptotics for the first collision time.

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