Electronic Communications in Probability > Vol. 5 (2000) > Paper 15 open journal systems

On Uniqueness of a Solution of Lu=u^a with Given Trace

Sergei E. Kuznetsov, *University of Colorado at Boulder*

Abstract

A boundary trace *(G, m)* of a solution of *Delta u = u^a* in a bounded smooth domain in *R^d* was first constructed by Le Gall who described all possible traces for *a = 2, d= 2* in which case a solution is defined uniquely by its trace. In a number of publications, Marcus, Veron, Dynkin and Kuznetsov gave analytic and probabilistic generalization of the concept of trace to the case of arbitrary *a > 1, d > 1*. However, it was shown by Le Gall that the trace, in general, does not define a solution uniquely in case *d >= (a +1)/(a -1)*. He offered a sufficient condition for the uniqueness and conjectured that a uniqueness should be valid if the singular part *G* of the trace coincides with the set of all explosion points of the measure *m*. Here, we establish a necessary condition for the uniqueness which implies a negative answer to the above conjecture.

Full text: PDF | PostScript

Pages: 137-147

Published on: May 7, 2000

Bibliography

1. C. Dellacherie and P.-A. Meyer, *Probabilités et potentiel*, Hermann, Paris, 1975, 1980, 1983, 1987. MR 58 #7557, MR 82b:60001, MR 86b:60003, MR 88k:60002
2. E. B. Dynkin, Superprocesses and partial differential equations, *Ann. Probab.* 21 (1993), 1185--1262. MR 94j:60156
3. E. B. Dynkin, Stochastic boundary values and boundary singularities for solutions of the equation *L u=u^alpha*, *J. Functional Analysis* 153 (1998), 147--186. MR 98m:60125
4. E. B. Dynkin and S. E. Kuznetsov, Linear additive functionals of superdiffusions and related nonlinear p.d.e., *Trans. Amer. Math. Soc.* 348 (1996), 1959--1987. MR 97d:60135
5. E. B. Dynkin and S. E. Kuznetsov, Solutions of *L u=u^alpha* dominated by *L*-harmonic functions, *Journale d'Analyse Mathematique* 68 (1996), 15--37. MR 97f:35048
6. E. B. Dynkin and S. E. Kuznetsov, Superdiffusions and removable singularities for quasilinear partial differential equations, *Comm. Pure & Appl. Math* 49 (1996), 125--176. MR 97m:60114
7. E. B. Dynkin and S. E. Kuznetsov, Fine topology and fine trace on the boundary associated with a class of quasilinear differential equations, *Comm. Pure & Appl. Math.* 51 (1998), 897--936. MR 99f:35046
8. E. B. Dynkin and S. E. Kuznetsov, Solutions of nonlinear differential equations on a {R}iemannian manifold and their trace on the Martin boundary, *Transact. Amer. Math. Soc.* 350 (1998), 4521--4552. MR 99c:60168c
9. E. B. Dynkin and S. E. Kuznetsov, Trace on the boundary for solutions of nonlinear differential equations, *Transact. Amer. Math. Soc.* 350 (1998), 4499--4519. MR 99a:60084
10. A. Gmira and L. Veron, Boundary singularities of solutions of some nonlinear elliptic equations, *Duke Math.J.* 64 (1991), 271--324. MR 93a:35053
11. S. E. Kuznetsov, *sigma*-moderate solutions of *Lu=u^alpha* and fine trace on the boundary, *C. R. Acad. Sci. Paris*, Serie I, 326 (1998), 1189--1194. MR 99g:35032
12. J.-F. Le Gall, Solutions positives de *Delta u=u^2* dans le disque unité, *C.R. Acad. Sci. Paris*, Serie I, 317 (1993), 873--878. MR 94h:35059
13. J.-F. Le Gall, A probabilistic approach to the trace at the boundary for solutions of a semilinear parabolic differential equation, *J. Appl.Math. Stochast. Analysis* 9 (1996), 399--414. MR 97m:35125

14. J.-F. Le Gall, A probabilistic Poisson representation for positive solutions of *Delta u = u^2* in a planar domain}}, *Comm. Pure & Appl Math.* (1997), 69--103. MR 98c:60144
15. M. Marcus and L. Veron, Trace au bord des solutions positives d'équations elliptiques non linéaires}, *C.R. Acad.Sci Paris*, Serie I, 321 (1995), 179--184. MR 96f:35045
16. M. Marcus and L. Veron, Trace au bord des solutions positives d'équations elliptiques et paraboliques non linéaires. Resultats d'existence and d'unicité, *C.R. Acad.Sci Paris*, Serie I, 323 (1996), 603--608. MR 97f:35012
17. M. Marcus and L. Veron, The boundary trace of positive solutions of semilinear elliptic equations, I: The subcritical case, *Arch. Rat. Mech. Anal.* 144 (1998), 201--231. MR 2000a:35077
18. M. Marcus and L. Veron, The boundary trace of positive solutions of semilinear elliptic equations: The supercritical case, *J. Math. Pures Appl.* 77 (1998), 481--524. MR 99g:35045

Electronic Communications in Probability. ISSN: 1083-589X