

On Subordinators, Self-Similar Markov Processes and Some Factorizations of the Exponential Variable

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Abstract

Let ξ be a subordinator with Laplace exponent Φ , $I = \int_0^\infty \exp(-\xi_s) ds$ the so-called exponential functional, and X (respectively, \hat{X}) the self-similar Markov process obtained from ξ (respectively, from $\hat{\xi} = -\xi$) by Lamperti's transformation. We establish the existence of a unique probability measure ρ on $]0, \infty[$ with k -th moment given for every $k \in \mathbb{N}$ by the product $\Phi(1) \cdots \Phi(k)$, and which bears some remarkable connections with the preceding variables. In particular we show that if R is an independent random variable with law ρ then IR is a standard exponential variable, that the function $t \mapsto E(1/X_t)$ coincides with the Laplace transform of ρ , and that ρ is the 1 -invariant distribution of the sub-markovian process \hat{X} . A number of known factorizations of an exponential variable are shown to be of the preceding form IR for various subordinators ξ .

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