On Recurrent and Transient Sets of Inhomogeneous Symmetric Random Walks

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Abstract

We consider a continuous time random walk on the *d*-dimensional lattice Z^d: the jump rates are time dependent, but symmetric and strongly elliptic with ellipticity constants independent of time. We investigate the implications of heat kernel estimates on recurrence-transience properties of the walk and we give conditions for recurrence as well as for transience: we give applications of these conditions and discuss them in relation with the (optimal) Wiener test available in the time independent context. Our approach relies on estimates on the time spent by the walk in a set and on a 0-1 law. We show also that, still via heat kernel estimates, one can avoid using a 0-1 law, achieving this way quantitative estimates on more general hitting probabilities.

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