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Error bounds on the non-normal approximation of Hermite power variations of fractional Brownian motion

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Abstract

Let $q \geq 2$ be a positive integer, B be a fractional Brownian motion with Hurst index $H \in (0,1)$, Z be an Hermite random variable of index q, and H_q denote the q-th Hermite polynomial. For any $n \geq 1$, set $V_n = \sum_{0 \leq k \leq n-1} H_q(B_{k+1} - B_k)$. The aim of the current paper is to derive, in the case when the Hurst index verifies $H > 1 - 1/(2q)$, an upper bound for the total variation distance between the laws of Z_n and of Z, where Z_n stands for the correct renormalization of V_n which converges in distribution towards Z. Our results should be compared with those obtained recently by Nourdin and Peccati (2007) in the case where $H < 1 - 1/(2q)$, corresponding to the case where one has normal approximation.

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