A dynamical characterization of Poisson-Dirichlet distributions

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Abstract

We show that a slight modification of a theorem of Ruzmaikina and Aizenman on competing particle systems on the real line leads to a characterization of Poisson-Dirichlet distributions PD(a,0). Precisely, let ξ be a proper random mass-partition i.e. a random sequence ($\xi_i, i \in N$) such that $\xi_1 >= \xi_2 >= ... >= 0$ and $\sum_i \xi_i = 1$ a.s.

Consider $\{W_i\}_{\{i \in N\}}$, an iid sequence of random positive numbers whose distribution is absolutely continuous with respect to the Lebesgue measure and E $[W^\lambda] < \infty$ for all $\lambda \in R$. It is shown that, if the law of ξ is invariant under the random reshuffling

($\xi_i, i \in N$) $\to$ ($\{\xi_i W_i\}/\{\sum_j \xi_j W_j\}, i \in N$)

where the weights are reordered after evolution, then it must be a mixture of Poisson-Dirichlet distributions PD(a,0), $a \in (0,1)$.

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Pages: 283-290

Published on: September 21, 2007

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